PRACTICE QUESTION PAPER - 1

CLASS – XII

Session : 2022 – 23

Subject : Mathematics

Time : 3 Hrs. **Max. Marks : 80**

General Instructions :

1. This question paper contains **FIVE SECTIONS – A, B, C, D and E.** Each section is compulsory. However, there are internal choices in some question.
2. **SECTION A** has **18 MCQ's** and **02 Assertion-Reason** based questions of **1 mark** each.
3. **SECTION B** has **5 Very Short Answer (VSA)-type** questions of **2 marks** each.
4. **SECTION C** has **6 Short Answer (SA)-type** questions of **3 marks** each.
5. **SECTION D** has **4 Long Answer (LA)-type** questions of **5 marks** each.
6. **SECTION E** has **3 source based/case based/passage based/integrated units of assessment** (**4 marks** each) with sub parts.

SECTION – A

1. If the inverse of the product of the matrix $B = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ with a matrix A is $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, then A^{-1} equals-

(a) $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 9 & 14 \\ 2 & 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 5 & 5 \\ 0 & 0 & 9 \\ 2 & 14 & 16 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 0 & 2 \\ 2 & 14 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -3 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

2. If $A = [a_{ij}]$ is a square matrix of even order such that $a_{ij} = i^2 - j^2$, then

(a) A is skew-symmetric matrix and |A| is a perfect square

(b) A is a skew-symmetric matrix and |A| = 0

(c) A is symmetric matrix and | A | = 0

(d) None of these

3. In a regular hexagon ABCDEF $\overrightarrow{AB}=\vec{a}, \overrightarrow{BC}=\vec{b}$ and $\overrightarrow{CD}=\vec{c}$. Then $\overrightarrow{AE}=$

(a) $\vec{a}+2\vec{b}+2\vec{c}$ (b) $2\vec{a}+\vec{b}+\vec{c}$ (c) $\vec{a}+\vec{b}+\vec{c}$ (d) $\vec{b}+\vec{c}$

4. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third component are 0.2, 0.3 and 0.5 respectively. What is the probability that the machine will fail?

(a) None of these (b) 0.07 (c) 0.072 (d) 0.70

5. $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx = ?$

(a) x cos 2α – sin 2α.log |sin(x + α)| + C (b) xcos2α+sin2αlog|sin(x+α)| + C

(c) x cos2α + sinα.log |sin(x + α)| + C (d) None of these

6. General solution of $\frac{dy}{dx}=(1+x^2)(1+y^2)$ is

(a) $\tan^{-1}y = x+\frac{x^3}{3}+C$ (b) $\cos^{-1}y = x+\frac{x^3}{3}+C$

(c) $\cot^{-1}y = x+\frac{x^3}{3}+C$ (d) $\sin^{-1}y = x+\frac{x^3}{3}+C$

7. The area bounded by the curves $y=\sqrt{x}, 2y+3=x$ and the x-axis in the first quadrant is

(a) 36 Sq. units (b) 18 Sq. units (c) 9 Sq. units (d) None of these

8. If $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=-\hat{i}-\hat{j}$ then the scalar components of vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=1$ is

(a) 1, 0, 0 (b) 0, 0, 1 (c) 0, –1, 0 (d) None of these

9. $\int_{\pi/6}^{\pi/3} \cot^2 x\,dx = ?$

(a) $-\frac{\pi}{6}+\frac{2}{\sqrt{3}}$ (b) $-\frac{\pi}{3}-\frac{2\sqrt{3}}{3}$ (c) $\frac{\pi}{6}+\frac{2}{\sqrt{3}}$ (d) $\frac{\pi}{3}+\frac{2\sqrt{3}}{3}$

10. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ (c) $\frac{d}{dx}(\Delta_1) = 2(\Delta_2)^2$ (d) $\Delta_1 = 3\Delta_2^{3/2}$

11. The area bounded by $y = 2\cos x$, $x = 0$ to $x = 2\pi$ and the axis of x in square units is-

(a) 4 (b) 6 (c) 8 (d) 7

12. The value of $\int_{-2}^{3} |1 - x^2|\, dx$ is

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

13. The point $(0, 5)$ is closest to the curve $x^2 = 2y$ at

(a) $(\sqrt{2}, 1)$ (b) $(\pm 2\sqrt{2}, 4)$ (c) $(2, 2)$ (d) (0, 0)

14. A biased die is tossed and the respective probabilities for various faces to turn up are given below

Face :	1	2	3	4	5	6
Probability :	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4, is

(a) 0.25 (b) 0.42 (c) 0.75 (d) 0.9

15. The sum of order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is.

(a) 1 (b) 2 (c) 3 (d) 4

16. The projection of any line on co-ordinate axes be respectively 3, 4, 5 then its length is

(a) 12 (b) 50 (c) $5\sqrt{2}$ (d) None of these

17. $\sec^2(\tan^{-1} 2) + \mathit{cosec}^2(\cot^{-1} 3)$ is:

(a) 5 (b) 13 (c) 15 (d) 6

18. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(-\frac{2}{3}\right)$ (c) None of these (d) $\cos^{-1}\left(\frac{3}{2}\right)$

19. **Assertion (A):** The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval $(2, \infty)$.

Reason (R): The function f(x) is strictly increasing in the interval (a, b) if $f'(x) > 0\ \forall\ x \in (a, b)$

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

20. **Assertion (A):** If $A=\begin{bmatrix}2 & -2\\ 4 & 3\end{bmatrix}$, then $A^{-1}=\frac{1}{14}\begin{bmatrix}3 & 2\\ -4 & 2\end{bmatrix}$

Reason (R): $A=\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}$, then $A^{-1}=\frac{1}{a_2b_1-a_1b_2}\begin{bmatrix}b_2 & -b_1\\ -a_2 & a_1\end{bmatrix}$

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

SECTION – B

21. Evaluate: $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\tan^{-1}(-\sqrt{3})+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

22. Find the general solution of y log y dx – x dy = 0.

23. Determine the values of x for which the matrix $A=\begin{bmatrix}x+1 & -3 & 4\\ -5 & x+2 & 2\\ 4 & 1 & x-6\end{bmatrix}$ is singular.

OR

Find the matrix X satisfying the equation: $\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}X\begin{bmatrix}5 & 3\\ 3 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

24. A line with direction cosines proportional to 2,1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$, then find the co-ordinates of each of the points of intersection.

25. An urn contains 5 white and 8 black balls. Two successive drawings of three balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

SECTION – C

26. Evaluate the integral: $\int (\sin^{-1} x)^3 dx$

27. If y(x) is a solution of the differential equation $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ and y(0) = 1, then find the value of $y\left(\frac{\pi}{2}\right)$.

OR

Find the particular solution of the differential equation $x\frac{dy}{dx} + 2y = x^2, (x \neq 0)$. Given that x = 2, y = 1.

28. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

OR

Verify that $\vec{a}\times(\vec{b}+\vec{c})=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$, when $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$.

29. Evaluate the integral: $\int \cos(\log_e x)\, dx$

OR

Evaluate: $\int \frac{1}{\sin x + \sec x} dx$

30. For what value of λ, the function defined by $f(x) = \begin{cases} \lambda(x^2+2) & , \text{ if } x \le 0 \\ 4x+6 & , \text{ if } x > 0 \end{cases}$ is continuous at x = 0 ?

Hence, check the differentiability of f(x) at x = 0.

31. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines x = 0 and x = 1.

SECTION – D

32. Solve the following LPP graphically:

Maximize Z = 5x + 7y

Subject to

$x + y \le 4$

$3x + 8y \le 24$

$10x + 7y \le 35$

$x, y \ge 0$

33. Let R be relation defined on the set of natural number N as follows:

$R = \{(x, y): x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

OR

Prove that the relation R on the set $N \times N$ defined by

$(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.

Also, find the equivalence classes [(2, 3)] and [(1, 3)].

34. Find the shortest distance between the lines given by $\vec{r} = (8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

OR

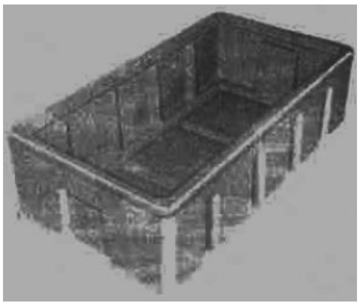
By computing the shortest distance determine whether the pairs of lines intersect or not:

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

35. If $e^y + xy = e$, then show that at $x = 0, \dfrac{d^2y}{dx^2} = \dfrac{1}{e^2}$

SECTION – E

36. Ram wants to construct a rectangular plastic tank for his house that can hold 100 ft^3 of water. The top of the tank is open. The width of tank will be 10 ft but the length and height are variables. Building the tank cost Rs. 20 per square foot for the base as well as for the sides.



Based on the above information, answer the following questions.

(i) Find the total cost of tank as a function of b

(ii). Find the interval in which the cost function C(b) is increasing or decreasing.

(iii). Find the total area of tank at which C(b) is minimum

OR

(iii). Find the cost of least expensive tank

37. Read the text carefully and answer the questions:

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

School	Article	Fans	Mats	Plates
A		40	50	20
B		25	40	30
C		35	50	40

i. Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.

ii. Find the funds collected by school A, B and C by selling the given articles.

iii. If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

iii. Find the total funds collected for the required purpose after 20% decrease in cost price of each unit..

38. Read the text carefully and answer the questions:

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

i. Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

ii. Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics ?