

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

Q.1 Let  $\vec{a}, \vec{b}$  be two vectors, and let  $P, Q$  and  $R$  be the points with position vectors  $\vec{a}, \vec{b}$  and  $\vec{a} + \vec{b}$ , respectively, with respect to the origin  $O$ . If  $|\vec{a} + \vec{b}| = \sqrt{21}$ ,  $|\vec{a} - \vec{b}| = 3$ , and  $\vec{a}$  and  $(\vec{a} - \vec{b})$  are perpendicular to each other, then the area of the triangle  $OPR$  is

(A)	$\sqrt{3}$	(B)	$\frac{\sqrt{3}}{2}$	(C)	$\frac{3\sqrt{3}}{2}$	(D)	$\frac{3}{2}$
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Q.2 Let  $T$  be the tangent to the parabola  $y^2 = 16x$  at the point  $(64, 32)$ . Let  $L$  be the tangent to the same parabola at another point  $(x_1, y_1)$  on the parabola. If  $L$  and  $T$  are perpendicular to each other, then the distance between the point  $(x_1, y_1)$  and the focus of the parabola, is

(A)	$\frac{15}{4}$	(B)	4	(C)	$\frac{17}{4}$	(D)	5
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Q.3 Let  $y : (-\infty, \infty) \rightarrow (0, \infty)$  be the solution of the differential equation

$$\frac{dy}{dx} = \frac{e^{5x}y^3 + y^3}{e^x + e^xy^4},$$

satisfying  $y(0) = \frac{1}{\sqrt{2}}$ . Then the value of  $y(\log_e 2)$  is

(A)	$\sqrt{\frac{5 + \sqrt{35}}{2}}$	(B)	$\sqrt{\frac{7 + \sqrt{53}}{2}}$
(C)	$\frac{7 + \sqrt{53}}{2}$	(D)	$\frac{5 + \sqrt{35}}{2}$

Q.4 The value of the definite integral

$$\int_0^2 \frac{1}{3^x + 3} dx$$

is

(A)	$\frac{1}{2}$	(B)	$\frac{1}{3}$	(C)	$\frac{\log_e 3}{3}$	(D)	$\frac{\log_e 3}{2}$
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**SECTION 2 (Maximum Marks: 20)**

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:
  - Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  - Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
  - Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  - Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  - Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  - Negative Marks* : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 mark;
  - choosing **ONLY** (B) will get +1 mark;
  - choosing **ONLY** (D) will get +1 mark;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -1 marks.

Q.5 Let  $\mathbb{R}$  denote the set of all real numbers. Consider the polynomial function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = \frac{d^{10}}{dx^{10}} ((x^2 - 1)^{10}), \quad \text{for all } x \in \mathbb{R}.$$

Here  $\frac{d^{10}}{dx^{10}} ((x^2 - 1)^{10})$  is the 10<sup>th</sup> order derivative of the function  $(x^2 - 1)^{10}$ .

Then which of the following statements is (are) TRUE ?

(A)	The coefficient of $x^8$ in the polynomial $f(x)$ is $(-10) \left( \frac{18!}{8!} \right)$
(B)	The value of $f(1) + f(-1)$ is equal to $10! 2^{11}$
(C)	The degree of the polynomial $f(x)$ is 10
(D)	The constant term of the polynomial $f(x)$ is $-\left( \frac{10!}{5!} \right)$

Q.6 Let  $a, b, c$  be positive integers in arithmetic progression such that the equation

$$ax^2 + bx + c = 0$$

has only integer solutions.

Then which of the following statements is (are) TRUE ?

(A)	$c - b$ is an integer multiple of $a$
(B)	Both the roots of the equation $ax^2 + bx + c = 0$ are odd integers
(C)	If $c = 15$ , then $ab = 8$
(D)	If $b = 8$ , then $x = 3$ is a root of the equation $ax^2 + bx + c = 0$

Q.7 Let  $L$  be the straight line joining the points  $P(1, 2, -1)$  and  $Q(2, 3, 1)$ . Let  $S$  be the foot of the perpendicular drawn from the point  $R(4, -1, 5)$  to the line  $L$ . Another line passing through  $R$  intersects  $L$  at a point  $T$  such that the point  $S$  divides the line segment  $PT$  internally in the ratio  $|PS| : |ST| = 1 : 2$ , where  $|PS|$  and  $|ST|$  are the lengths of the line segments  $PS$  and  $ST$ , respectively.

Then which of the following statements is (are) TRUE ?

(A)	The orthocentre of the triangle $PRT$ is $(\frac{23}{5}, -4, \frac{31}{5})$
(B)	The orthocentre of the triangle $PRT$ is $(4, 3, 5)$
(C)	The area of the triangle $PRT$ is $6\sqrt{5}$
(D)	The area of the triangle $PRT$ is $18\sqrt{5}$

Q.8 Let  $y = f(x)$  be the real valued function defined on the interval  $(0, \infty)$ , satisfying  $y(1) = 0$  and the differential equation

$$x \frac{dy}{dx} = y - x^3.$$

Then which of the following statements is (are) TRUE ?

(A)	The function $f$ has a local minimum at $x = \frac{1}{\sqrt{3}}$
(B)	The function $f$ has a local maximum at $x = \frac{1}{\sqrt{3}}$
(C)	The function $f$ is increasing in the interval $(1, 2)$
(D)	If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for $x > 0$ , then the number of elements in the set $\{x \in (0, \infty) : f(x) = g(x)\}$ is 2

Q.9

Let  $\mathbb{R}$  denote the set of all real numbers and let  $i = \sqrt{-1}$ . Consider the matrices

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Let  $a, b, c, d$  be real numbers such that

$$ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Let

$$H = \{x + iy : x, y \in \mathbb{R} \text{ and } y > 0\}.$$

Then which of the following statements is (are) TRUE ?

(A)	$\frac{b+ia}{d+ic} = i$
(B)	If $\omega = \frac{-1+i\sqrt{3}}{2}$ , then $\frac{a\omega+b}{c\omega+d} = \omega$
(C)	If $m$ is an integer greater than 2 such that $(ST)^2 = (ST)^m$ , then $m$ is an integer multiple of 8
(D)	If $z \in H$ , then $\frac{az+b}{cz+d} \in H$

**SECTION 3 (Maximum Marks: 20)**

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:  
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*Zero Marks* : 0 In all other cases.

Q.10 Let  $\mathbb{N}$  denote the set of all positive integers. Consider the sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{1, 2, 3, 4, 5, 6, 7\}.$$

Let  $S$  be the set of all functions  $f: A \rightarrow B$  such that  $f(2) \neq 2$  and  $f(4) \neq 4$ . Consider the set

$$T = \{f \in S : \text{there exists a function } g: B \rightarrow \mathbb{N} \text{ such that } g(f(x)) = 2^x \text{ for all } x \in A\}.$$

Then the number of elements in the set  $T$  is \_\_\_\_\_.

Q.11 A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let  $X$  be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If  $\alpha$  is the mean of the random variable  $X$ , then the value of  $77\alpha$  is \_\_\_\_\_.

Q.12 Consider a data consisting of 10 observations  $x_1, x_2, \dots, x_{10}$ , whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations  $x_1, x_2, \dots, x_8$  are 4 and 3.5, respectively, and  $x_9 < x_{10}$ , then the value of  $3x_9 + 2x_{10}$  is \_\_\_\_\_.

Q.13 Consider the ellipse  $E$  given by  $\frac{x^2}{18} + \frac{y^2}{12} = 1$ . Let  $H$  be the hyperbola whose eccentricity is the reciprocal of the eccentricity of  $E$  and whose foci are the same as that of  $E$ . Let  $P$  and  $Q$  be the points of intersection of  $H$  and the parabola  $\sqrt{5}y = x^2$  in the first quadrant. Let  $d$  be the distance between  $P$  and  $Q$ .

If  $a$  and  $b$  are the integers such that  $d^2 = a + b\sqrt{5}$ , then the value of  $a - b$  is \_\_\_\_\_.

Q.14 For a real number  $\alpha$ , let  $[\alpha]$  denote the greatest integer less than or equal to  $\alpha$ . For a finite set  $S$ , let  $|S|$  denote the number of elements in the set  $S$ .

Consider the functions  $f : (-3, 3) \rightarrow (-\infty, \infty)$  and  $g : (-3, 3) \rightarrow (-\infty, \infty)$  defined by

$$f(x) = [x^3] \log_e(1 + \sin^2(\pi(x - [x])))$$

and

$$g(x) = x^3 \sin^2(\pi \log_e(1 + x - [x])).$$

Let

$$A = \{x \in (-3, 3) : f \text{ is discontinuous at } x\}$$

and

$$B = \{x \in (-3, 3) : g \text{ is discontinuous at } x\}.$$

Then the value of  $|A| + 2|B| - |A \cap B|$  is \_\_\_\_\_.

**SECTION 4 (Maximum Marks: 8)**

- This section contains **TWO (02)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
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**Question Stem for Question Nos. 15 and 16**

Consider the curve  $C_1$  given by

$$y = e^{-x} \quad \text{for } x \in [0, 10\pi],$$

and the curve  $C_2$  given by

$$y = e^{-x}(\sin x + \cos x) \quad \text{for } x \in [0, 10\pi].$$

Let  $n$  be the total number of points of intersection of the curves  $C_1$  and  $C_2$ .

Suppose that  $\alpha_1, \alpha_2, \dots, \alpha_n \in [0, 10\pi]$  are the  $x$ -coordinates of the points of intersection of the curves  $C_1$  and  $C_2$  such that

$$\alpha_1 < \alpha_2 < \dots < \alpha_n.$$

Q.15 The value of  $n$  is \_\_\_\_\_.

Q.16 Let  $\beta$  be the area of the region enclosed between the curves  $C_1$ ,  $C_2$ , and the lines  $x = \alpha_1$  and  $x = \alpha_4$ . Then the value of

$$-\frac{1}{\pi} \log_e \left( \beta - 2 e^{-\frac{\pi}{2}} \right)$$

is \_\_\_\_\_.

**Question Stem for Question Nos. 17 and 18**

Consider the ellipses given by

$$x^2 + 4y^2 = 1 \quad \text{and} \quad 4x^2 + y^2 = 1 .$$

Q.17 Let  $P$  be the point in the first quadrant where the given ellipses intersect. If  $\theta$  is the acute angle between the tangents to the given ellipses at the point  $P$ , then the value of  $4 \tan \theta$  is \_\_\_\_\_.

Q.18 If  $\alpha$  is the area of the common region that lies inside both the given ellipses, then the value of  $\cot \alpha$  is \_\_\_\_\_.

**END OF THE QUESTION PAPER**

**SECTION 1 (Maximum Marks: 12)**

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- Q.1 A metal wire of cross-sectional area  $0.5 \text{ mm}^2$  and length  $100 \text{ m}$  is connected across a battery of e.m.f.  $2 \text{ V}$  and internal resistance  $1 \Omega$ . The density, atomic mass and electrical conductivity of the metal are  $6.35 \times 10^3 \text{ kg m}^{-3}$ ,  $63.5 \text{ gm/mole}$  and  $2 \times 10^8 \text{ mho m}^{-1}$ , respectively. Assuming one conduction electron per atom of the metal, the drift velocity (in  $\text{mm s}^{-1}$ ) of the electrons in the wire is:  
 [Take Avogadro's number as  $6 \times 10^{23}$  and charge of the electron as  $1.6 \times 10^{-19} \text{ C}$ .]

(A)	0.052	(B)	0.104	(C)	0.208	(D)	0.156
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- Q.2 A nuclear reactor starts producing a radioactive nuclide  $X$  from  $t = 0$ , at a constant rate of  $\alpha$  per second. Each decay of  $X$  produces energy  $E_0$ , which is utilized to heat a liquid of mass  $m$  and specific heat  $s$ . Assuming no heat loss from the liquid and taking  $\lambda$  as the decay constant of  $X$ , the rate of increase in the temperature of the liquid is:

(A)	$\frac{\alpha E_0}{m s} (1 - e^{-\lambda t})$	(B)	$\frac{\alpha E_0}{m s} (e^{\lambda t} - 1)$
(C)	$\frac{\lambda E_0}{m s} (1 - e^{-\lambda t})$	(D)	$\frac{E_0}{m s} (\alpha - \lambda e^{-\lambda t})$

- Q.3 A beam of polychromatic light passes through a thin prism of prism angle  $6^\circ$ . The refractive index of the material of the prism varies with wavelength ( $\lambda$ ) as  $n(\lambda) = \alpha\lambda + \frac{\beta}{\lambda^2}$ , where  $\alpha = 3 \mu\text{m}^{-1}$  and  $\beta = 0.096 \mu\text{m}^2$ . If  $\lambda_{\min}$  is the wavelength at which the angle of minimum deviation  $D_m$  is smallest, then the correct value of  $D_m$  at  $\lambda_{\min}$  is

(A)	$6.4^\circ$	(B)	$4.8^\circ$	(C)	$3.2^\circ$	(D)	$2.4^\circ$
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Q.4

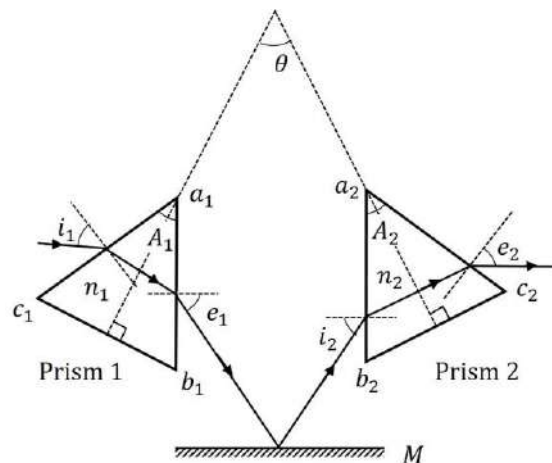
A particle of mass  $m$ , and angular momentum  $\ell$  is moving in a circular orbit of radius  $r_0$  under the influence of an attractive force  $\vec{F}(r) = -\frac{k}{r^2}\hat{r}$ . Keeping its angular momentum unchanged, the particle is displaced radially by a small distance  $\delta r \ll r_0$ , due to which its radial distance varies periodically. The corresponding time period is:

(A)	$\frac{2\pi\ell^3}{mk^2}$	(B)	$2\pi\sqrt{\frac{m}{k}}$	(C)	$\frac{2\pi\ell^3}{3mk^2}$	(D)	$\frac{2\pi\ell^3}{5mk^2}$
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  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
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  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 mark;
  - choosing **ONLY** (B) will get +1 mark;
  - choosing **ONLY** (D) will get +1 mark;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -1 marks.

- Q.5 Consider two isosceles prisms 1 and 2 with prism angles  $A_1$  and  $A_2$  and refractive indices  $n_1$  and  $n_2$ , respectively, as shown in the figure. The faces  $a_1b_1$  and  $a_2b_2$  are parallel to each other and perpendicular to the mirror  $M$ . If a ray of light is incident on the face  $a_1c_1$  and emerges from the face  $a_2c_2$ , then the correct statement(s) is/are:



- |     |   |
|-----|---|
| (A) | If both the prisms are at minimum deviation condition, then $\frac{n_2}{n_1} = \sin\left(\frac{A_1}{2}\right) / \sin\left(\frac{A_2}{2}\right)$ .   |
| (B) | If prism 2 is at minimum deviation condition, then $\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ is always true.  |
| (C) | If both the prisms 1 and 2 are thin and are at minimum deviation condition with angles of deviation $\delta_{m1}$ and $\delta_{m2}$ , respectively, then $\theta = \frac{\delta_{m1}}{2(n_1-1)} + \frac{\delta_{m2}}{2(n_2-1)}$ . |
| (D) | If prism 1 is at minimum deviation condition, then $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ is always true.  |

- Q.6 In a vacuum chamber, a particle of charge  $1 \mu\text{C}$  and mass  $1 \text{ mg}$  is projected with a velocity  $(\hat{i} + 2\hat{j}) \text{ ms}^{-1}$  from the  $XZ$  plane at time  $t = 0$  in an electric field of  $1\hat{i} \text{ Vm}^{-1}$ . At  $t = 0.2 \text{ s}$ , the electric field is switched off and a magnetic field of  $6\hat{j} \text{ T}$  is switched on. The acceleration due to gravity is  $-10\hat{j} \text{ ms}^{-2}$ . Correct option(s) is/are:

- |     |   |
|-----|---|
| (A) | The vertical distance of the particle from the $XZ$ plane at $t = 0.3 \text{ s}$ is $15 \text{ cm}$ . |
| (B) | The vertical distance of the particle from the $XZ$ plane at $t = 0.4 \text{ s}$ is $10 \text{ cm}$ . |
| (C) | The radius of the trajectory of the particle for $t > 0.2 \text{ s}$ is $20 \text{ cm}$ .             |
| (D) | The particle will be in the $XZ$ plane at $t = 0.35 \text{ s}$ .                                      |

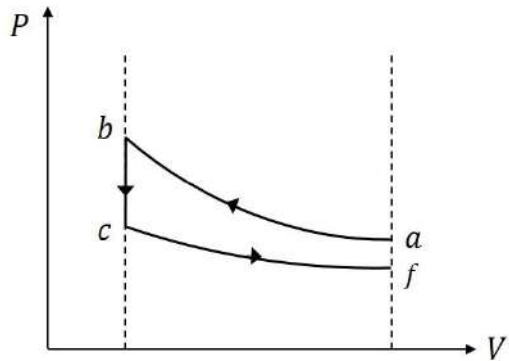
- Q.7 Two charges  $Q_1 = q$  and  $Q_2 = mq$  are placed at the points  $P_1(a, b)$  and  $P_2(ma, mb)$ , respectively, in the  $XY$  plane, where  $a, b \neq 0$  and  $m \neq 0, 1$ . If  $V_1$  is the potential at a point in the  $XY$  plane due to charge  $Q_1$  and  $V_2$  is the potential at that point due to charge  $Q_2$ . Correct statement(s) for the points at which  $|V_1| = |V_2|$  is/are:

- |     |   |
|-----|---|
| (A) | For $m = -1$ , locus of these points is $ax + by = 0$ .   |
| (B) | For $m = 2$ , the locus of these points is a circle of radius $\frac{2}{3}\sqrt{a^2 + b^2}$ centered at $\left(\frac{2}{3}a, \frac{2}{3}b\right)$ |
| (C) | For $m = -2$ , the locus of these points is a circle of radius $2\sqrt{a^2 + b^2}$ centered at $(2a, 2b)$   |
| (D) | For $m = -3$ , locus of these points is $3bx + 3ay = 0$ .   |

Q.8 Consider an electric dipole comprising two charges  $+q$  and  $-q$  each with mass  $m$ , separated by a fixed distance  $d$  and initially at rest with its dipole moment pointing along  $\hat{i}$ . A uniform electric field  $E\hat{j}$  is turned on at time  $t = 0$  and it is turned off at  $t = t_f$ , when the dipole moment makes an angle  $\theta_f$  with  $\hat{i}$ . Neglecting any sources of energy loss, correct option(s) is/are:

- |     |  |
|-----|--|
| (A) | The center of mass of the dipole is deflected towards $\hat{j}$ in the presence of the field.                            |
| (B) | If the magnitude of the final angular velocity $\omega_f = \sqrt{\frac{2qE}{md}}$ , then $\theta_f = \frac{\pi}{6}$ .    |
| (C) | If $\theta_f = \pi/3$ , then the change in kinetic energy of the dipole is given by $2\sqrt{3} qEd$ .                    |
| (D) | For $\theta_f = \pi/4$ , the dipole rotates around its center of mass with a constant angular velocity after $t > t_f$ . |

Q.9 Ten moles of an ideal monoatomic gas, initially in state **a** at atmospheric pressure and temperature  $T_a = 27^\circ\text{C}$ , is enclosed in a metal cylinder of volume  $V_0$  fitted with a frictionless piston. The gas is suddenly compressed to state **b** with volume  $V_0/3$ . Now, keeping the piston stationary, the cylinder is submerged in a water bath of temperature  $11^\circ\text{C}$  until the gas reaches the temperature of the water bath, which is denoted as state **c**. Finally, while still in the water bath, the piston is brought slowly to its initial position, which is denoted as state **f**. If  $R$  is universal gas constant, then the correct option(s) is/are:  
[Given:  $9^{1/3} = 2.08$ ]

- |     |  |
|-----|--|
| (A) | The schematic P-V diagram of the processes described above is :<br> |
| (B) | The change in internal energy in going from state <b>a</b> to <b>b</b> is $4860R$ .  |
| (C) | The net change in the internal energy in the whole process is $-240R$ .  |
| (D) | The pressure and temperature of the state <b>b</b> are 2.08 times the atmospheric pressure and 624 K, respectively.                                    |

**SECTION 3 (Maximum Marks: 20)**

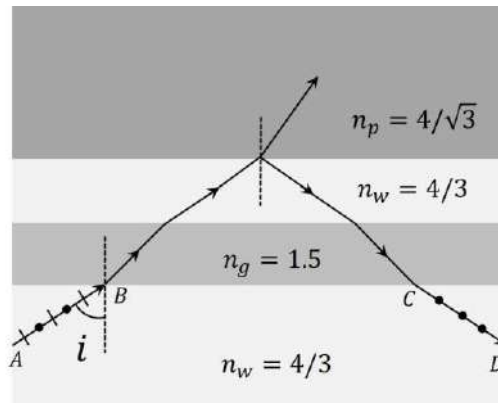
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*Full Marks* : +4 If **ONLY** the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

- Q.10 Two thin wires, Wire-1 of diameter 0.650 mm and Wire-2 of unknown diameter  $d$  are given. To obtain the value of  $d$ , the diameters of the two wires are measured with a screw gauge. The screw gauge has a pitch of 0.5 mm and there are 100 divisions on the circular scale (CS). The smallest division on the linear scale (LS) is 0.5 mm. The table shows the readings of LS and CS for the measurements. The value of  $d$  (in  $\mu\text{m}$ ) is:

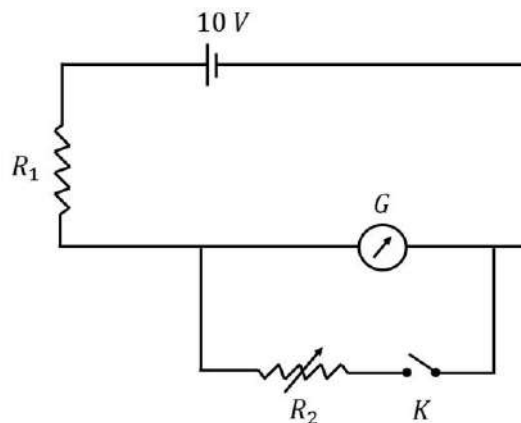
	Readings	
	LS (mm)	CS
Wire-1	0.5	42
Wire-2	1.5	95

- Q.11 In a single slit diffraction experiment, a slit of width  $(0.016 \pm 0.002)$  mm is used to measure the wavelength of a monochromatic light source. In the diffraction pattern, the angular distance between the central maximum and first minimum is measured to be  $(2^\circ \pm 40')$ . The value of the fractional error in the measurement of wavelength is:  
 [Given:  $\sin(2^\circ) = 0.035$ ]

- Q.12 As shown in the figure, a ray  $AB$  of unpolarized light enters from water of refractive index  $n_w = 4/3$  into a medium of refractive index  $n_p = 4/\sqrt{3}$  after passing through a glass plate of refractive index  $n_g = 1.5$  and a layer of water. At a particular incident angle  $i$  the reflected ray  $CD$  is polarized in the direction as shown in the figure. The value of  $i$  (in degrees) is:



- Q.13 As shown in the figure, the resistance of a galvanometer  $G$  can be found by the half-deflection method. Here the resistance  $R_2$  is adjusted such that when the key  $K$  is closed the deflection in the galvanometer becomes half of the value as compared to when  $K$  is open. Half-deflection is obtained at  $R_2 = 4 \Omega$  and thus the galvanometer resistance is found to be  $6 \Omega$ . In this half-deflection condition the current (in mA) through the resistor  $R_1$  is:



- Q.14 In a new system of units, the units of mass, length, time and current are 5 kg, 5 m, 5 s and 5 A, respectively. If  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of free space, respectively, then in this new system of units, the magnitude of one SI unit of  $\sqrt{\mu_0/\epsilon_0}$ , is:

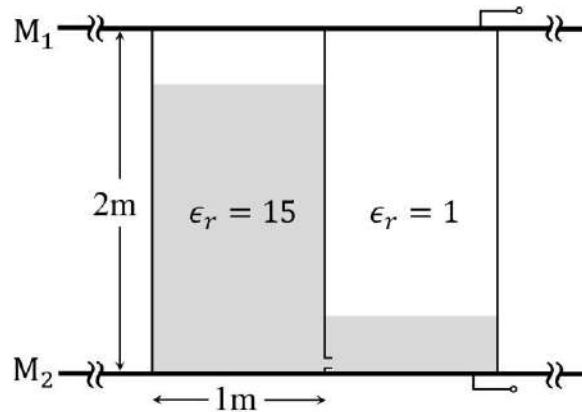
**SECTION 4 (Maximum Marks: 8)**

- This section contains **TWO (02)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**  
*Full Marks* : +2 If **ONLY** the correct numerical value is entered at the designated place;  
*Zero Marks* : 0 In all other cases.

### Question Stem for Question Nos. 15 and 16

A container of height 2 m, length 2 m and breadth 1 m is made of insulating vertical walls and two large area horizontal metal plates ( $M_1$  and  $M_2$ ) which extend far beyond the vertical walls in all directions. The container is partitioned into two equal chambers with a thin insulating vertical wall. The partition wall contains a small hole of cross-sectional area  $\sqrt{10}$  cm<sup>2</sup> near its bottom edge. Initially the hole is closed and the left chamber of the container is completely filled with a liquid of dielectric constant  $\epsilon_r = 15$  and the right chamber is empty ( $\epsilon_r = 1$ ). At time  $t = 0$ , the hole is opened and the liquid flows from the left chamber to the right chamber. In both the chambers, the space above the liquid has  $\epsilon_r = 1$  and is maintained at atmospheric pressure. The schematic of the container at a time  $t > 0$  is shown in the figure.

[Given: acceleration due to gravity is  $10 \text{ ms}^{-2}$ .]



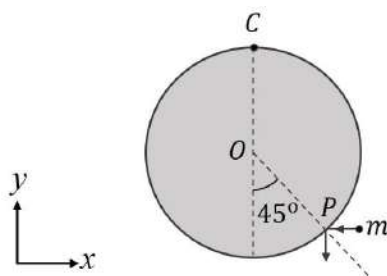
Q.15 The height (in m) of the liquid in left chamber at  $t = 500$  s is:

Q.16 The difference in the capacitance (in F) between the metal plates at  $t = 0$  and that at  $t = 500$  s is  $(8 - n)\epsilon_0$ , where  $\epsilon_0$  is the permittivity of free space. The value of  $n$  is:

**Question Stem for Question Nos. 17 and 18**

A uniform circular disk of radius 0.2 m and mass 1 kg is pivoted at its top point  $C$  such that it can rotate freely around  $C$  in the  $XY$  plane, as shown in the figure. Initially, when the disk is at rest, a particle of mass 20 g, travelling along negative  $x$  direction in the  $XY$  plane with speed  $100 \text{ ms}^{-1}$ , hits the circumference of the disk at a point  $P$ . After collision the particle moves along negative  $y$  direction at a speed of  $90 \text{ ms}^{-1}$ .

[Given: the acceleration due to gravity ( $g$ ) =  $-10 \hat{j} \text{ ms}^{-2}$ ]



Q.17 After the collision the disk starts to rotate around point  $C$  in the  $XY$  plane. The maximum change in the height (in m) of its center  $O$  is:

Q.18 Amount of energy loss (in J) in the collision is:

**END OF THE QUESTION PAPER**

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

- Q.1 At 300 K, the molar conductivities of the aqueous solutions of three salts at two different concentrations are given below:

Salt	Concentration (M)	Molar conductivity (S cm <sup>2</sup> mol <sup>-1</sup> )
NaNO <sub>3</sub>	0.01	111
	0.04	101
NaCl	0.01	117
	0.04	107
AgNO <sub>3</sub>	0.01	125
	0.04	116

The conductivity of a saturated aqueous solution of AgCl is  $1.40 \times 10^{-6}$  S cm<sup>-1</sup> at 300 K. If the solubility of AgCl in water at 300 K is  $X$  mol L<sup>-1</sup>, then  $\log_{10}(X^{-1})$  is

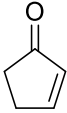
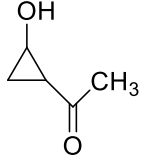
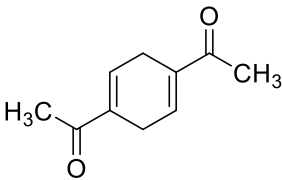
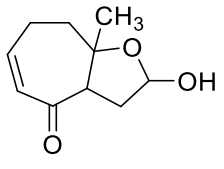
(Assume that AgCl dissolved in water ionizes completely and that the molar conductivity of saturated AgCl solution is equal to its limiting molar conductivity.)

(A)	3
(B)	4
(C)	5
(D)	6

- Q.2 The correct order of ONO bond angle in the given species is

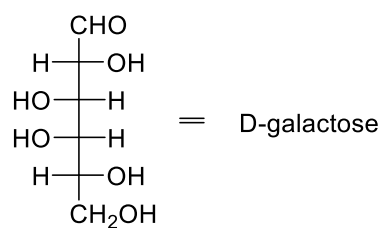
(A)	$\text{NO}_2^+ < \text{NO}_2 < \text{NO}_3^- < \text{NO}_2^-$
(B)	$\text{NO}_2^- < \text{NO}_3^- < \text{NO}_2 < \text{NO}_2^+$
(C)	$\text{NO}_3^- < \text{NO}_2^- < \text{NO}_2 < \text{NO}_2^+$
(D)	$\text{NO}_2^- < \text{NO}_3^- < \text{NO}_2^+ < \text{NO}_2$

Q.3 Natural rubber on complete ozonolysis ( $O_3/Zn-H_2O$ ) gives compound **X** as the major product. **X** gives positive iodoform and Tollen's tests. **X** on heating with aqueous NaOH gives **Y** as the major product. **Y** is

(A)		(B)	
(C)		(D)	

Q.4

A known artificial sweetener **X** is composed of 4-chloro-4-deoxy- $\alpha$ -D-galactose and 1,6-dichloro-1,6-dideoxy- $\beta$ -D-fructose joined by a glycosidic linkage. Structure of D-galactose is given below:



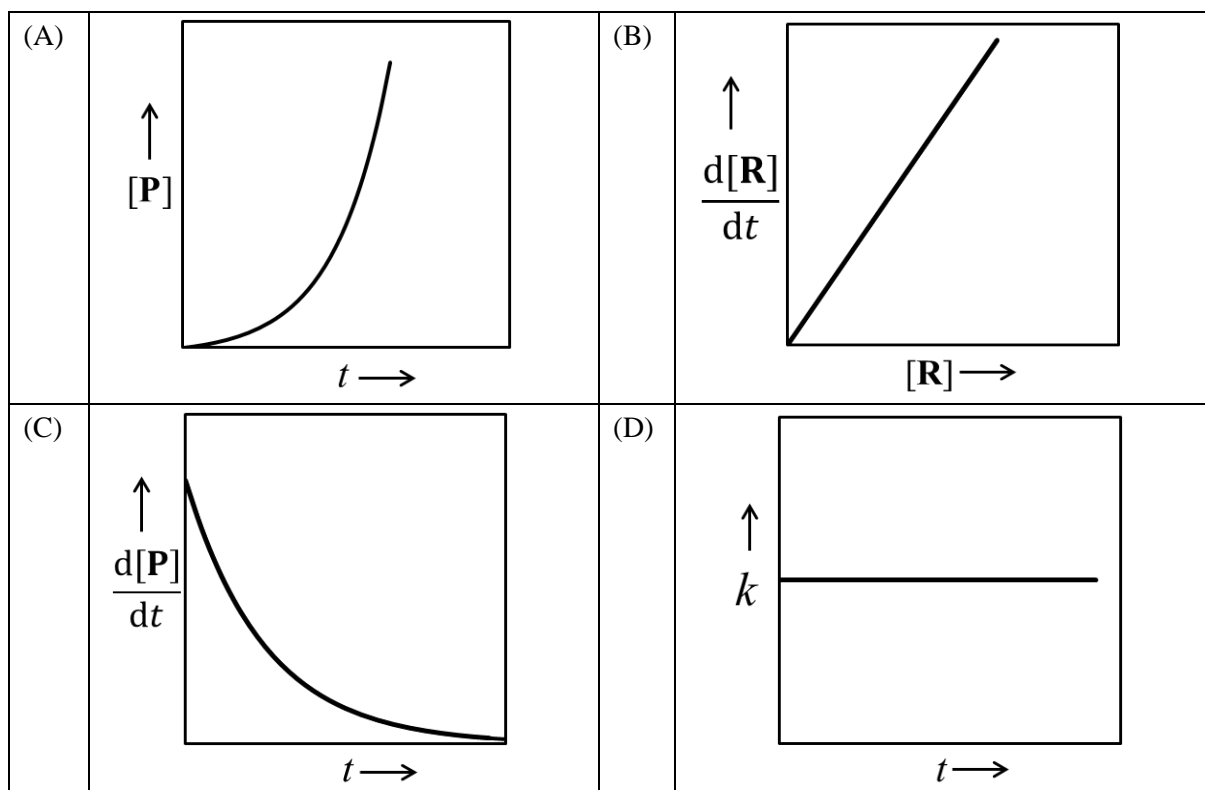
The correct structure of **X** is

(A)	
(B)	
(C)	
(D)	

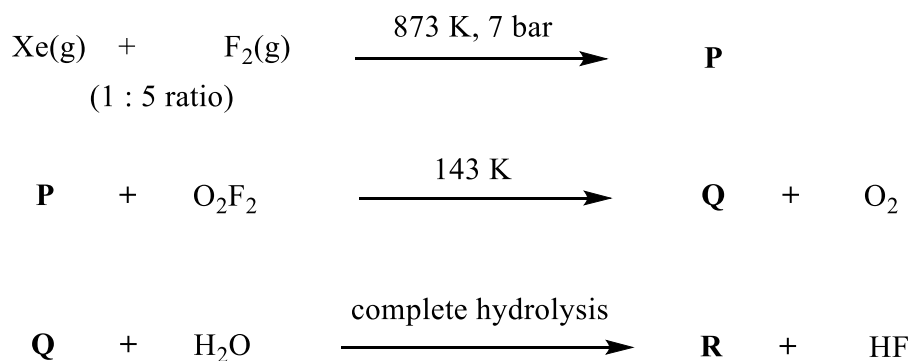
**SECTION 2 (Maximum Marks: 20)**

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
  - Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  - Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
  - Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  - Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  - Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  - Negative Marks* : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 mark;
  - choosing **ONLY** (B) will get +1 mark;
  - choosing **ONLY** (D) will get +1 mark;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -1 marks.

- Q.5 For a first-order reaction  $\mathbf{R} \rightarrow \mathbf{P}$  at a given temperature,  $k$  is the rate constant. For this reaction, at the given temperature, the concentrations of  $\mathbf{R}$  and  $\mathbf{P}$  at a time  $t$  are  $[\mathbf{R}]$  and  $[\mathbf{P}]$ , respectively. The correct graphical representation(s) for this reaction is(are)



- Q.6 Correct statement(s) about the compounds  $\mathbf{P}$ ,  $\mathbf{Q}$  and  $\mathbf{R}$  is(are)

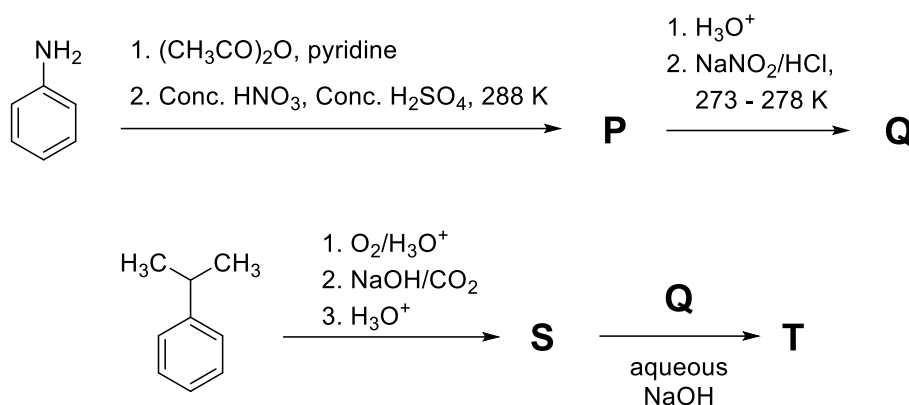


(A)	$\mathbf{P}$ has two lone pairs of electrons on the central atom.
(B)	$\mathbf{Q}$ has a perfect octahedral geometry.
(C)	$\mathbf{Q}$ can act as a fluorinating agent.
(D)	The molecular structure of $\mathbf{R}$ is trigonal pyramidal.

Q.7 The correct statement(s) regarding the periodic properties of elements is(are)

(A)	Second ionization enthalpy of carbon atom is less than that of boron atom.
(B)	Increasing order of ionic radii: $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+$
(C)	Under identical conditions, in solid state, the density of potassium metal is more than density of sodium metal.
(D)	The H–H bond is weaker than F–F bond.

Q.8 In the following reaction sequence, **P**, **Q**, **S** and **T** are the major products.



The correct statement(s) about **P**, **Q**, **S** and **T** is(are)

(A)	<b>Q</b> on treatment with ethanol generates an aromatic aldehyde.
(B)	<b>S</b> gives positive phthalein dye test.
(C)	<b>P</b> is a dinitro compound.
(D)	<b>T</b> is a coloured compound.

Q.9 The correct statement(s) regarding sugars is(are)

**Given:** Specific rotations of L-(–)-glucose and L-(+)-fructose are  $-52.5^\circ$  and  $+92.5^\circ$ , respectively.

(A)	On treatment with $\text{HNO}_3$ , gluconic acid is oxidized to saccharic acid, whereas glucose is not oxidized to saccharic acid.
(B)	Fructose gives a positive Fehling's test because it isomerises to glucose and another aldohexose in the presence of Fehling's reagent.
(C)	Invert sugar is an equimolar mixture of D-glucose and D-fructose formed after hydrolysis of the corresponding disaccharide.
(D)	Specific rotation of invert sugar is $-40^\circ$ .

**SECTION 3 (Maximum Marks: 20)**

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +4 If **ONLY** the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

Q.10  $X^{a+}$  and  $Y^{b+}$  are hydrogen-like species. The wavelength of light absorbed during the transition between the states with principal quantum numbers  $n = 1$  and  $n = 2$  of  $X^{a+}$  is  $\lambda$ . The wavelength of light absorbed during the transition between the states with principal quantum numbers  $n = 2$  and  $n = 4$  of  $Y^{b+}$  is  $9\lambda$ . The lowest possible value of  $(a+b)$  is \_\_\_\_.

Q.11 At a given temperature, 0.45 g of acetic acid in 50 mL of water is shaken with 1.0 g of charcoal and the pH of the resulting solution is 3.0. Assume, the adsorption of acetic acid from the aqueous solution by charcoal follows Freundlich isotherm,

$$\frac{x}{m} = kC^{1/n}$$

If the plot of  $\log_{10}(x/m)$  against  $\log_{10}C$  gives a straight line with slope 1, the value of  $k$  in  $\text{L mol}^{-1}$  is \_\_\_\_.

**Given:** The molar mass of acetic acid is  $60 \text{ g mol}^{-1}$ .

The acid dissociation constant of acetic acid is  $1.0 \times 10^{-5}$  at the given temperature.

$x$  is the mass (in grams) of acetic acid adsorbed.

$m$  is the mass (in grams) of charcoal.

$C$  is the equilibrium concentration of acetic acid in the solution after the adsorption is complete.

$k$  and  $n$  are constants for acetic acid–charcoal system at the given temperature.

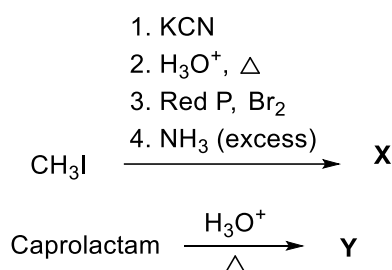
Q.12 In a solvent **S**, a compound **B** is partially dissociated into **C** and **D** as given below:



**B**, **C** and **D** are non-volatile in nature. The molar mass of **B** is 10 times the molar mass of **S**. The standard boiling point and the standard enthalpy of vaporization of **S** are 400 K and  $10R \text{ J mol}^{-1}$ , respectively ( $R$  is the gas constant in  $\text{J K}^{-1} \text{ mol}^{-1}$ ). A solution of **B** in **S** with an initial concentration of **B** as 0.25% (mass/mass) has a boiling point of 408 K at 1 bar pressure. In this solution, the mole percent of **B** that has been dissociated is \_\_\_\_.

Q.13 Consider that the coordinating atoms of the ligands in *cis*- $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$  and *mer*- $[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$  octahedral complexes are at the vertices of an octahedron. The sum of total number of the triangular faces in both the complexes having one N atom and two Cl atoms at their corners is \_\_\_\_.

Q.14 In the following reaction sequence, major products **X** and **Y** are acyclic monomers.



500 mol of **X** completely reacts with 500 mol of **Y** to give 1 mol of a single biodegradable acyclic copolymer **Z** as the only product. The amount of **Z** formed in grams is \_\_\_\_.

**Given:**

Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, Br : 80

**SECTION 4 (Maximum Marks: 8)**

- This section contains **TWO (02)** question stems.
- This section contains **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**  
*Full Marks* : +2 If **ONLY** the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

**Question Stem for Question Nos. 15 and 16**

Two volatile liquids **A** and **B** form an ideal solution. Consider a 5 molal solution of **B** in **A** inside a closed container having a total vapour pressure of 100 mm Hg at 300 K. The vapour pressure of pure **A** at 300K is 105 mm Hg. Assume that **A** and **B** behave as ideal gases in the vapour phase.

**Given:**

The gas constant  $R = 0.08 \text{ L atm K}^{-1} \text{ mol}^{-1}$

Molar mass of **A** is  $50 \text{ g mol}^{-1}$

Molar mass of **B** is  $57 \text{ g mol}^{-1}$

Density of liquid **B** at 300 K is  $0.5 \text{ g/mL}$

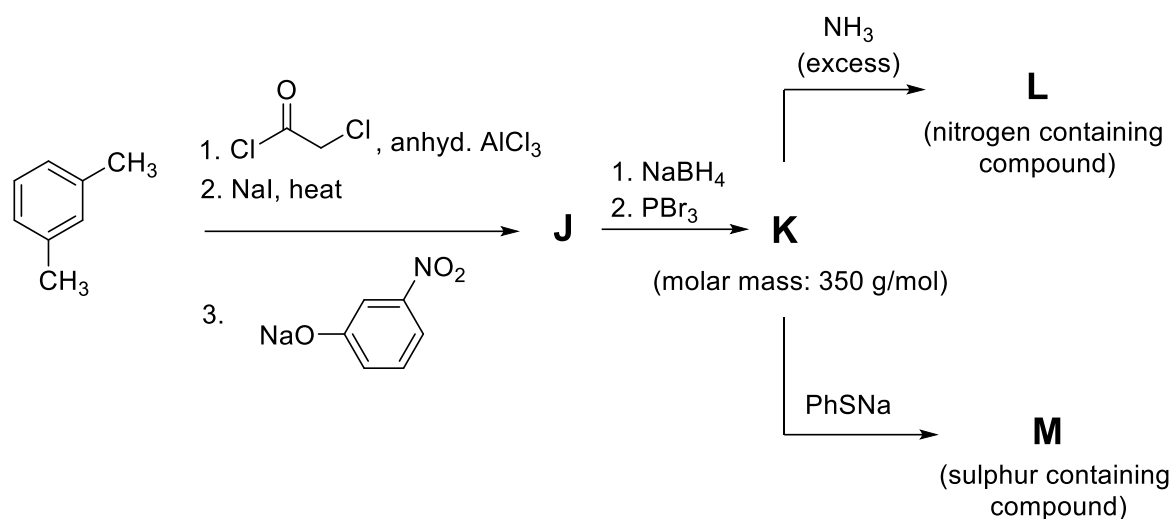
$1 \text{ atm} = 760 \text{ mm Hg}$

Q.15 At 300 K, the ratio of the molar volume of pure **B** in vapour phase to its molar volume in liquid phase is \_\_\_\_.

Q.16 The mole fraction of **B** in vapour phase which is in equilibrium with this solution is \_\_\_\_.

**Question Stem for Question Nos. 17 and 18**

Consider the following reaction sequence in which **J**, **K**, **L** and **M** are the major products.



**Given:**

Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, S : 32, Br : 80, Ba : 137

Q.17 The volume of 1 M aqueous H2SO4 required to completely neutralize the ammonia evolved from 5.72 g of **L** in Kjeldahl's method of nitrogen estimation is \_\_\_\_ mL.

Q.18 In sulphur estimation by Carius method, the amount of BaSO4 formed from 3.79 g of **M** is \_\_\_\_ g.

**END OF THE QUESTION PAPER**