JEE(ADVANCED)–2026 (EXAMINATION)

(Held On Sunday 17th MAY, 2026)

MATHEMATICS | **TEST PAPER WITH ANSWER AND SOLUTION**

PAPER-1

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

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1. Consider the function $f : (0, \infty) \to (-\infty, \infty)$ given by

$f(x) = \sqrt{x}\log_e(x) - x + 1$

Then which one of the following statements is TRUE ?

(A) The derivative of the function f is decreasing in the interval (0, 1)

(B) The function f has a local maximum at some point $a \in (0, \infty)$

(C) The function f has a local minimum at some point $b \in (0, \infty)$

(D) The function f has NEITHER a point of local maximum NOR a point of local minimum in the interval $(0, \infty)$

Ans. (D)

Sol. $f(x) = \sqrt{x}\,\ell nx - x + 1$

$$f'(x) = \sqrt{x}.\frac{1}{x} + \ell nx.\frac{1}{2\sqrt{x}} - 1$$

$$f'(x) = \frac{\ell nx + 2}{2\sqrt{x}} - 1$$

$$f''(x) = \frac{2\sqrt{x}\left(\frac{1}{x}\right) - (\ell nx + 2).\frac{1}{\sqrt{x}}}{4x}$$

$$= \frac{2 - \ell nx - 2}{4x\sqrt{x}} = \frac{-\ell nx}{4.x^{3/2}}$$

If f' is decreasing

$f''(x) \le 0$

$$\frac{-\ell nx}{4x^{3/2}} \le 0 \Rightarrow \ell n x \ge 0 \Rightarrow x \ge 1 \Rightarrow x \in (1, \infty)$$

So, options A is wrong.

Now, $f'(x) = \dfrac{\ell nx + 2 - 2\sqrt{x}}{2\sqrt{x}}$

$\because$ f'(1) = 0 ($\therefore$ x = 1 is critical point)

$x \in (0,1)$

$\sqrt{x} < 1$ and $\ell nx < 0$

$g(x) = \ell nx + 2 - 2\sqrt{x}$; $g(1) = 0$

$g'(x) = \dfrac{1}{x} - \dfrac{2}{2\sqrt{x}} = \dfrac{1-\sqrt{x}}{x}$

$x \in (0,1)$

$g'(x) > 0$

g(x) is inc.

x < 1

g(x) < g(1)

$\ell nx + 2 - 2\sqrt{x} < 0$

$x \in (1,\infty)$

$g'(x) < 0$

g(x) is dec.

x > 1

g(x) < g(1)

$\ell nx + 2 - 2\sqrt{x} < 0$

$\Rightarrow$ for $x \in (0,\infty)$

$\ell nx + 2 - 2\sqrt{x} < 0$

$\therefore f'(x) = \dfrac{\ell nx + 2 - 2\sqrt{x}}{2\sqrt{x}} < 0 \quad \forall x \in (0,\infty)$

$\therefore$ f(x) is decreasing $\forall x \in (0,\infty)$

It has no local maxima and no local minima

option (D) is correct.

2. Let P be the point on the parabola $y = x^2$ such that the slope of the tangent to the parabola at the point P is 4. Let Q be the point in the first quadrant lying on the circle $x^2 + y^2 = 2$ such that the slope of the tangent to the circle at the point Q is –1. Let R be the point in the first quadrant lying on the ellipse $x^2 + 4y^2 = 8$ such that the slope of the tangent to the ellipse at the point R is $-\frac{1}{2}$. Then the radius of the circle passing through the points P, Q and R is

(A) $\sqrt{10}$ (B) $\sqrt{5}$ (C) $\sqrt{\frac{5}{2}}$ (D) $2\sqrt{5}$

Ans. **(C)**

Sol. $y = x^2$

$\frac{dy}{dx} = 2x = 4 \Rightarrow x = 2$ (slope at point P is 4)

$\Rightarrow P(2, 4)$

$x^2 + y^2 = 2 \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

Slope at point Q is – 1

$\therefore -\frac{x}{y} = -1 \Rightarrow x = y$

Since Q lies in first quadrant

$x^2 + x^2 = 2 \Rightarrow x = 1$ and $y = x = 1$

$\Rightarrow Q (1, 1)$

$x^2 + 4y^2 = 8$

$2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$

$\because$ Slope of tangent at point R is $-\frac{1}{2}$

$\Rightarrow -\frac{x}{4y} = -\frac{1}{2} \Rightarrow x = 2y$

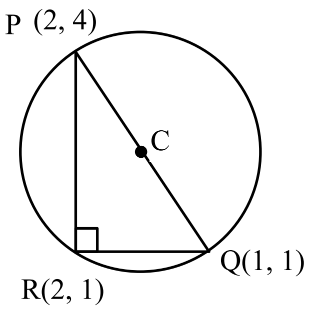
Since R is lies in first quadrant

$(2y)^2 + 4y^2 = 8 \Rightarrow 8y^2 = 8 \Rightarrow y = 1$

$x = 2y = 2$

$\Rightarrow R(2, 1)$

Find the equation of circle passing through P, Q and R



Since PQR is a Right angle triangle so radius of circle is $\frac{\sqrt{10}}{2} = \sqrt{\frac{5}{2}}$

3. Which one of the following matrices can be obtained by performing elementary row transformations on the 3 × 3 identity matrix ?

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$

Ans. **(B)**

Sol. A matrix can be obtained from 3 × 3 identity matrix by elementary row transformation iff it is non-singular matrix.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = -2 \neq 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix} = 0$$

Option (B) is correct

4. Considering only the principal values of the inverse trigonometric functions, the value of

$$\cot^{-1}(\cot(-11)) + 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10\sin(2\tan^{-1}(2))$$

is

(A) $3\pi + 7$ (B) 7

(C) $4\pi + 7$ (D) $3\pi - 5$

Ans. **(C)**

Sol. $\cot^{-1}(\cot(-11)) + 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10\sin\left(2\tan^{-1}2\right)$

$= 4\pi - 11 + 10\sin(\pi/2) + 10\sin\left(2\tan^{-1}2\right)$

$(\because 4\pi - 11 \in (0, \pi))$

$= 4\pi - 11 + 10 + 10\sin\left(2\tan^{-1}2\right)$

Let $\tan^{-1}2 = \theta$ $\left(\theta \in \left(0, \frac{\pi}{2}\right)\right)$

$\Rightarrow \tan\theta = 2$

So $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{4}{5}$

$\Rightarrow$ Ans. $= 4\pi - 11 + 10 + 10 \times \frac{4}{5} = 4\pi + 7$

SECTION-2 : (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –1 marks.

5. Suppose that Box I contains 6 red balls and 9 green balls, and Box II contains 8 red balls and 12 green balls. All the balls of Box I and Box II are mixed together and a ball is chosen at random from them. Let E_1 be the event that the ball chosen belonged to Box I and let E_2 be the event that the ball chosen belonged to Box II. Let F_1 be the event that the ball chosen is red and let F_2 be the event that the ball chosen is green.

Then which of the following statements is (are) TRUE ?

(A) The events E_1 and F_1 are independent

(B) The events E_2 and F_2 are dependent

(C) The conditional probability $P(F_1|E_1)$ is equal to the conditional probability $P(F_1|E_2)$

(D) The conditional probability $P(F_1|E_1)$ is greater than the conditional probability $P(F_2|E_2)$

Ans. **(A,C)**

Sol.

Box I	Box II
6R	8R
9G	12G

Total Red balls = 14

Total Green balls = 21

Total balls = 35

$$P(E_1) = \frac{15}{35} = \frac{3}{7}$$

$$P(E_2) = \frac{20}{35} = \frac{4}{7}$$

$$P(F_1) = \frac{14}{35} = \frac{2}{5}$$

$$P(F_2) = \frac{21}{35} = \frac{3}{5}$$

Now, $P\left(\frac{F_1}{E_1}\right) = \frac{6}{15} = \frac{2}{5}$

$P\left(\frac{F_1}{E_2}\right) = \frac{8}{20} = \frac{2}{5}$

Option (C) is correct

$P\left(\frac{F_2}{E_2}\right) = \frac{12}{20} = \frac{3}{5}$

$\because P\left(\frac{F_1}{E_1}\right) > P\left(\frac{F_2}{E_2}\right)$

Option D is incorrect

Since $P\left(\frac{F_1}{E_1}\right) = \frac{2}{5}$ and $P(F_1) = \frac{2}{5}$

$P\left(\frac{F_1}{E_1}\right) = P(F_1)$

So E_1 and F_1 are not independent.

Option (A) is correct

And $P\left(\frac{F_2}{E_2}\right) = \frac{3}{5}$ and $P(F_2) = \frac{3}{5}$

$P\left(\frac{F_2}{E_2}\right) = P(F_2)$

So E_2 and F_2 are not dependent events.

Option (B) is incorrect

6. Let P be the plane such that it contains the straight line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{1}$ and is perpendicular to the plane $x + 2y + 3z = 4$. Let P_1 be the plane which passes through the point (4, 2, 2) and is parallel to P. Then which of the following statements is (are) TRUE ?

(A) The equation of the plane P is $7x - 5y + z = -10$

(B) The distance between the planes P and P_1 is 30

(C) The distance of the plane P from the origin is $2\sqrt{3}$

(D) The acute angle between the plane P and the plane $2x + 2y + z = 3$ is $\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

Ans. **(A,D)**

Sol. Since plane P contains the line

$L:\dfrac{x-1}{2}=\dfrac{y-3}{3}=\dfrac{z+2}{1}$ and perpendicular to the plane x + 2y + 3z = 4

So normal vector of plane

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & 2 & 3\end{vmatrix}$$

$$\vec{n}=7\hat{i}-5\hat{j}+\hat{k}$$

Since line passes through (1, 3, –2)

So plane P is also passes through if

7(x – 1) – 5(y – 3) + 1 (z + 2) = 0

Plane P: 7x – 5y + z = – 10

Option A is correct

Since plane P_1 is parallel to plane P

So 7x – 5y + z = d

We are given that P_1 passes through the point (4, 2, 2)

So 7(4) – 5(2) + 2 = d ⇒ d = 20

⇒ Plane P_1: 7x – 5y + z = 20

Distance between plane P and P_1 = $\left|\dfrac{d_1-d_2}{\sqrt{A^2+B^2+C^2}}\right|=\left|\dfrac{-10-20}{\sqrt{49+25+1}}\right|=\dfrac{30}{\sqrt{75}}$

Option B is incorrect

Distance from (0, 0, 0) to plane P = $\dfrac{10}{\sqrt{75}}=\dfrac{2}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

Option C is incorrect

Acute angle between plane

P & 2x + 2y + z = 3

$$\cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{7\times2+(-5)\times2+1\times1}{\sqrt{75}\times3}=\frac{1}{3\sqrt{3}}$$

$$\theta=\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

Option D is correct.

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be an arbitrary function and let $g : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$(x) = (x) \to x\ f(x), \text{ for all } x \in \mathbb{R}.$$

Then which of the following statements is (are) TRUE ?

(A) The function g is always continuous at $x = 0$

(B) If f is continuous at $x = 0$, then g is differentiable at $x = 0$

(C) If g is differentiable at $x = 0$, then f is continuous at $x = 0$

(D) If g is differentiable at $x = 0$, then $\lim\limits_{x\to 0} f(x)$ exists.

Ans. **(B,D)**

Sol. (A) Suppose $f(x) = \begin{cases} \frac{1}{x}\sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

$$g(x) = xf(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

which is not continuous at x = 0

(B) $f(0^-) = f(0^+) = f(0)$

$$g'(0) = \lim_{h\to 0}\frac{g(h)-g(0)}{h}, \quad \because g(0) = 0$$

$$g'(0) = \lim_{h\to 0}\frac{g(h)}{h} = \lim_{h\to 0}\frac{hf(h)}{h}$$

$g'(0) = \lim\limits_{h\to 0} f(h) = f(0)$, which is finite since f(x) is continuous at x = 0.

$\therefore$ g(x) is differentiable at x = 0

(C) $g'(0) = \lim\limits_{h\to 0} f(h)$,

$\because$ g(x) is given as differentiable at x = 0,

we have $\lim\limits_{h\to 0} f(h)$ existing

but nothing can be said about f(0).

Hence, we cannot comment on continuity of f(x) at x = 0 just because g(x) is differentiable at x = 0.

(D) It correct as proven in option (C)

8. Consider the matrix

$$M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

Let p, q, r, s, a, b, c and d be integers such that

$$M^{26} = \begin{bmatrix} p & q \\ r & S \end{bmatrix} \text{ and } \sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then which of the following statements is (are) TRUE ?

(A) There exists a 2 × 2 invertible matrix N with real entries such that

$$MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(B) The value of a is 378

(C) For any two given integers m and n, there exist unique integers x and y such that

$$px + qy = m \text{ and } rx + sy = n$$

(D) For each positive real number t, the system of linear equations

$$(a + t)x + by = 1$$

$$cx + (d + t)y = -1$$

has a unique solution

Ans. **(A,C,D)**

Sol. $M^2 = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$

$M^3 = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$

By observation:

$M^k = \begin{bmatrix} k+1 & -k \\ k & 1-k \end{bmatrix}$

$2 + 3 + 4 + \ldots + 27 = \frac{26}{2}(29)$

$\therefore\ M^{26} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$\Rightarrow p = 27 ; q = -26 ; r = 26 ; s = -25$

$\sum_{k=1}^{26} M^k = \begin{bmatrix} \Sigma k + 1 & -\Sigma k \\ \Sigma k & \Sigma 1 - k \end{bmatrix} = \begin{bmatrix} 377 & -351 \\ 351 & -325 \end{bmatrix}$

$\Rightarrow a = 377 ; b = -351 ; c = 351 ; d = -325$

Option A:

$$MN = N\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

Let $N = \begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix} = \begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix}2\alpha-\gamma & 2\beta-\delta\\ \alpha & \beta\end{bmatrix} = \begin{bmatrix}\alpha & \alpha+\beta\\ \gamma & \gamma+\delta\end{bmatrix}$

$\Rightarrow$ $2\alpha - \gamma = \alpha$; $2\beta - \delta = \alpha + \beta$

$\alpha = \gamma$; $\beta = \gamma + \delta$

$\Rightarrow$ $\alpha = \gamma$; $\beta = \alpha + \delta$

$\therefore$ N can be $\begin{bmatrix}\alpha & \alpha+\delta\\ \alpha & \delta\end{bmatrix}$

where $\alpha, \delta \in R$

$\therefore$ A is correct

Option B is wrong

Option C :

system : $27x - 26y = m$

$26x - 25y = n$

$$\Delta = \begin{vmatrix}27 & -26\\ 26 & -25\end{vmatrix} = -675 + 676 = 1$$

$\because$ $D \neq 0 \Rightarrow$ consistent system with unique solution

$\therefore$ C is correct

Option D :

system :

$a = 377$; $b = -351$; $c = 351$; $d = -325$

$(377 + t)x - 351y = 1$

$351x + (t - 325)y = -1$

$$\Delta = \begin{vmatrix} 377+t & -351 \\ 351 & t-325 \end{vmatrix}$$

$\Rightarrow$ $\Delta = (377 + t)(t - 325) + (351)^2$

$= t^2 + 52t + (351)^2 - (377)(325)$

$= t^2 + 52t + (351)^2 - (351+26)(351-26)$

$= t^2 + 52t + (351)^2 - ((351)^2 - (26)^2)$

$= t^2 + 52t + (26)^2$

$= (t + 26)^2$

$\because$ t is +ve $\Rightarrow \Delta > 0\ \forall$ real positive t

$\therefore$ system is consistent

$\Rightarrow$ (D) is correct

SECTION-3 : (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer using in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

9. Let S = {1, 2, 3, …, 10}. Consider the set

X = {R : R is an equivalence relation on the set S such that R has exactly 42 elements}.

Then the number of elements in X is __________.

Ans. 2520.00

Sol. The total number of such relations is 2520.

Here is how you can think about it step-by-step :

An equivalence relation is just a way of breaking a group into smaller, separate teams (called equivalence classes). Within any team, every member connects to everyone else, including themselves.

This means a team with 'n' people creates n^2 total connections.

For our problem, we have 10 people in total. We need to split them into teams so that the sum of the squared team sizes equals exactly 42.

We need to find combinations of numbers that add up to 10, but whose squares add up to 42. If you test out different numbers, only two combinations actually work :

Option A : One team of b, one team of 2 and two teams of 1. Check total people : (6 + 2 + 1 + 1 = 10) check total connections : ($6^2 + 2^2 + 1^2 + 1^2 = 36 + 4 + 1 + 1 = 42$).

Option B : One team of 5, one team of 4 and one team of 1. Check total people : (5 + 4 + 1 = 10) Check total connections : ($5^2 + 4^2 + 1^2 = 25 + 16 + 1 = 42$).

Now we just count how many ways we can sort our 10 people into these two setups.

Fo option A(6, 2, 1, 1) : First, pick 6 people out of 10 for the big team.

Then, pick 2 out of the remaining 4 for the second team. The last 2 people automatically form their own single-peson teams.

Because the two single person teams are identical in size, we divide by 2 to avoid double-counting them.

$$\text{Ways} = \frac{10!}{6!\,2!\,1!\,1!} \times \frac{1}{2!} = 1260$$

For option B

Pick 5 people out of 10 fo the first team, then 4 out of the remaining 5 for the second team.

The last person is left by themselves.

$$\text{Ways} = \frac{10!}{5!\,4!\,1!} = 1260$$

Total count

Add the tow possibilities together : (1260 + 1260 = 2520)

10. Consider the function $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to (-\infty, \infty)$ defined by

$$f(x) = (|x| + |x-1|)\sin x + [x \sin x],$$

where [x sin x] is the greatest integer less than or equal to x sin x.

Let α be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is **NOT** continuous, and let β be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is **NOT** differentiable.

Then the value of $\alpha + \beta$ is __________.

Ans. 5.00

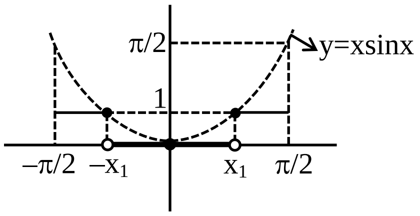
Sol. Observe [x sinx]

let g(x) = xsinx

$g'(x) = \sin x + x\cos x \geq 0$

$g'(x) \geq 0 \ \forall \ x \in \left[0, \frac{\pi}{2}\right)$

$g'(x) \le 0 \ \forall \ x \in \left(-\frac{\pi}{2}, 0\right]$



[xsinx] discontinuous at $x = x_1, -x_1$; $x_1 \in \left(0, \frac{\pi}{2}\right)$

[xsinx] not diff. at $x = x_1, -x_1, x_1 \in \left(0, \frac{\pi}{2}\right)$

Now $y = (|x| + |x - 1|)$ sinx

(i) continuous $\forall$ x $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ii) Not differentiable at x = 1, differentiable at x = 0

Here $x_1 \ne 1$

Now

(i) for diffrentiability

$$f(x) = \underbrace{(|x|+|x-1|)\sin x}_{f_1(x)} + \underbrace{[x\sin x]}_{f_2(x)}$$

at $x = x_1, -x_1$

$f_1(x)$ is differentiable & $f_2(x)$ is not differentiable

Hence f(x) is not differentiable

at x = 1 ; $f_1(x)$ is not differentiable & $f_2(x)$ is differentiable

$\Rightarrow$ f(x) is not differentiable

at x = 0

$f_1(x)$ & $f_2(x)$ both differentiable
Hence $\beta = 3$
(ii) for continuity, we can clearly say
at $x = x_1, -x_1$,
f(x) will be discontinuous
$\alpha = 2$

$\boxed{\alpha + \beta = 5}$

11. The number of ways to distribute 10 identical red pens and 14 identical blue pens among four persons such that each person gets 6 pens, is ____________.

Ans. **206.00**

Sol. Let r_i & b_i are number of pens recieved by i^{th} person

$\therefore\ r_i + b_i = 6$

& $r_1 + r_2 + r_3 + r_4 = 10$

also $b_i = 6 - r_i$

$\because\ r_i \geq 0$ and $b_i \geq 0$

$\Rightarrow r_i \leq 6$

$\therefore\ 0 \leq r_i \leq 6$

$\therefore$ we have to find possible cases of

$r_1 + r_2 + r_3 = r_4 = 10,\ \ 0 \leq r_i \leq 6$

$\therefore$ Required cases $= {}^{10+4-1}C_3 - [{}^4C_1 \times {}^6C_3]$

$= {}^{13}C_3 - 4 \times {}^6C_3$

$= 286 - 80$

$= 206$

12. Let

$$\alpha = \left(1-2\cos\left(\frac{\pi}{11}\right)\right)\left(1-2\cos\left(\frac{3\pi}{11}\right)\right)\left(1-2\cos\left(\frac{9\pi}{11}\right)\right)\left(1-2\cos\left(\frac{27\pi}{11}\right)\right)\left(1-2\cos\left(\frac{81\pi}{11}\right)\right).$$

Then the value of $5 - \alpha^2$ is ____________.

Ans. **4.00**

Sol. Let $\theta = \frac{\pi}{11}$ then

$a = (1 - 2\cos\theta)\ (1 - 2\cos3\theta)\ (1 - 2\cos9\theta)\ ...\ (1 - 2\cos81\theta)$

$\because\ 1 - 2\cos\theta$

$$= 1 - 2\left(2\cos^2\frac{\theta}{2} - 1\right)$$

$$= 3 - 4\cos^2\frac{\theta}{2}$$

$$= \frac{\left(3 - 4\cos^2\frac{\theta}{2}\right)\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$= -\frac{\left(4\cos^3\frac{\theta}{2} - 3\cos\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}}$$

$$= -\frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\therefore\ \alpha = \left(-\frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}}\right) \times \left(-\frac{\cos\left(\frac{9\theta}{2}\right)}{\cos\frac{3\theta}{2}}\right) \times \ldots \left(-\frac{\cos\frac{243\theta}{2}}{\cos\frac{81\theta}{2}}\right)$$

$$\alpha = -\frac{\cos\frac{243\theta}{2}}{\cos\frac{\theta}{2}}$$

Now $\frac{243\theta}{2} = \frac{243\pi}{22} = 11\pi + \frac{\pi}{22}$

$$\Rightarrow \alpha = \frac{-\cos\left(11\pi + \frac{\pi}{22}\right)}{\cos\left(\frac{\pi}{22}\right)} = 1$$

$\boxed{\because 5 - \alpha^2 = 4}$ Ans.

SECTION-4 : (Maximum Marks : 16)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

13. Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I		List-II	
(P)	If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2026}}$ and $\frac{1}{(\beta+1)^{2026}}$ is	(1)	$x^2 + x + 1 = 0$
(Q)	If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2027}}$ and $\frac{1}{(\beta+1)^{2027}}$ is	(2)	$x^2 - x + 1 = 0$
(R)	If γ and δ are the distinct roots of the equation $x^2 - x + 1 = 0$, then the value of $\frac{1}{(\gamma-1)^{2026}} + \frac{1}{(\delta-1)^{2026}}$ is	(3)	$x^2 + x - 1 = 0$
(S)	If p and r are the distinct roots of the equation $x^2 + x - 1 = 0$, then the value of $\frac{1}{(p+1)^3} + \frac{1}{(r+1)^3}$ is	(4)	-1
		(5)	-4

(A) (P)→(1), (Q)→(2), (R)→(5), (S)→(4)

(B) (P)→(3), (Q)→(1), (R)→(4), (S)→(5)

(C) (P)→(1), (Q)→(2), (R)→(4), (S)→(5)

(D) (P)→(2), (Q)→(3), (R)→(5), (S)→(4)

Ans. **(C)**

Sol. (P) α,β are roots of $x^2 + x + 1 = 0$

$\therefore\ \alpha = w\ ,\ \beta = w^2$

$(\omega + 1)^{2026} = (-\omega^2)^{2026} = (\omega^2)^{2025}.\ \omega^2$

$= w^2$

$(\omega^2 + 1)^{2026} = (-\omega)^{2026} = \omega$

Equation whose roots are $\frac{1}{(\alpha+1)^{2026}}$ and $\frac{1}{(\beta+1)^{2026}}$

$x^2 + x + 1 = 0\ \therefore\ P \to 1$

(Q) $(\omega + 1)^{2027} = (-\omega^2)^{2027} = -(\omega^2)^{2025}.(\omega^2)^2$

$= -\omega$

$(\omega^2 + 1)^{2027} = (-\omega^2)^{2027} = -w^2$

$x^2 + (\omega + \omega^2)\ x + w^3 = 0$

$x^2 - x + 1 = 0$

(R) Roots of $x^2 - x + 1 = 0$

$x = -\omega,\ x = -\omega^2$

$(\gamma - 1)^{2026} + (\delta - 1)^{2026}$

$(\omega + 1)^{2026} + (\omega^2 + 1)^{2026}$

$\Rightarrow \omega^2 + \omega = -1$

(S) $p + r = -1$ $\quad$ $pr = -1$

$p^2 + r^2 + 2pr = 1$

$p^2 + r^2 - 2 = 1\ \therefore\ p^2 + r^2 = 3$

$p^3 + r^3 + 3pr\ (p + r) = -1$

$p^3 + r^3 - 3(-1) = -1\ \therefore\ p^3 + r^3 = -4$

$(p + 1)\ (r + 1) = pr + p + r + 1$

$= -1 - 1 + 1$

$= -1$

$$\frac{1}{(p+1)^3}+\frac{1}{(r+1)^3}=\frac{(p+1)^3+(r+1)^3}{[(p+1)(r+1)]^3}$$

$$=\frac{p^3+r^3+2+3p^2+3p+3r^2+3r}{(-1)^3}$$

$$\equiv\frac{-4+2+3(3)+3(-1)}{-1}=-4$$

14. Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I		List-II	
(P)	The number of elements in the set $\{x\in[-\pi,\pi]:\sin^6 x+\cos^4 x=1\}$	(1)	is 1
(Q)	The number of elements in the set $\left\{x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]:\sin^2 x+\cos^6 x=1\right\}$	(2)	is 2
(R)	The number of elements in the set $\left\{x\in[-\pi,\pi]:\cos^2\left(\frac{x}{2}\right)-\sin^2 x=\frac{1}{2}\right\}$	(3)	is 3
(S)	The number of elements in the set $\left\{x\in[-2\pi,2\pi]:6\sin^2\left(\frac{x}{2}\right)-\cos 3x=3\right\}$	(4)	is 4
		(5)	is 5

(A) (P)→(2), (Q)→(5), (R)→(3), (S)→(4)

(B) (P)→(5), (Q)→(3), (R)→(2), (S)→(4)

(C) (P)→(5), (Q)→(4), (R)→(1), (S)→(3)

(D) (P)→(4), (Q)→(3), (R)→(2), (S)→(5)

Ans. **(B)**

Sol. (P) $\sin^6x+\cos^4x=1$

$\sin^2x\le 1$; $\cos^2x\le 1$

$\sin^6x\le\sin^2x$ $\cos^4x\le\cos^2x$

$\sin^6x+\cos^4x=1$ only if

$\sin^6x=\sin^2x$ and $\cos^4x=\cos^2x$

$\sin^2x=0$ and $\cos^2x=0$

or $\sin^4x=1$ $\cos^2x=1$

$$x=-\pi,0,\pi,-\frac{\pi}{2},\frac{\pi}{2}$$

(Q) $\sin^2 x + \cos^6 x = 1$

$\sin^2 x = 1\ \cos^2 x = 0$

or $\sin^2 x = 0\ \ \cos^2 x = 1$

$x = \frac{\pi}{2}, 0, \frac{\pi}{2}$

(R) $\cos^2\frac{x}{2} - \sin^2 x = \frac{1}{2}$

$\frac{1+\cos x}{2} - (1-\cos^2 x) = \frac{1}{2}$

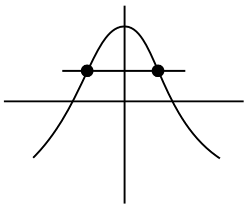
$\Rightarrow 1 + \cos x - 2 + 2\cos^2 x = 1$

$2\cos^2 x + \cos - 2 = 0$

$\cos x = \frac{-1 \pm \sqrt{1+16}}{4}$

$\cos x = \frac{-1+\sqrt{17}}{4}, \frac{-1-\sqrt{17}}{4}$

only possibility $\cos x = \frac{-1+\sqrt{17}}{4}, \frac{-1-\sqrt{17}}{4}$



Number of elements in $[-\pi, \pi]$ is 2

(S) $6\sin^2\frac{x}{2} - \cos 3x = 3$

$-\cos 3x = 3\left(1 - 2\sin^2\frac{x}{2}\right)$

$-\left[4\cos^3 x - 3\cos x\right] = 3[\cos x]$

$4\cos^3 x = 0$

$x = \frac{-3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

in $[-2\pi, 2\pi]$ 4 solutions

15. For real numbers α, β, γ, δ and μ, consider the matrix

$$M = \begin{bmatrix} \alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}} \\ \gamma & \delta & \mu \end{bmatrix}$$

Suppose that $MM^T = I$, where M^T is the transpose of the matrix M, and I is the 3 × 3 identity matrix.

Let $\vec{u} = \alpha\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \gamma\hat{k}$, $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \beta\hat{j} + \delta\hat{k}$ and $\vec{w} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \mu\hat{k}$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List – I		**List – II**
(P)	The value of $\gamma^2 + \delta^2$ is	(1)	0
(Q)	If $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$ for some real numbers x, y and z, then the value of x is	(2)	1
(R)	The value of $\left\vert\vec{u}.\left(\vec{v}\times\vec{w}\right)\right\vert$ is	(3)	$\frac{1}{\sqrt{2}}$
(S)	The value of $\left\vert\vec{u}\times\left(\vec{v}\times\vec{w}\right)\right\vert$ is	(4)	$\frac{1}{\sqrt{3}}$
		(5)	$\frac{5}{6}$

(A) (P) → (5), (Q) → (4), (R) → (2), (S) → (1)

(B) (P) → (4), (Q) → (5), (R) → (1), (S) → (2)

(C) (P) → (5), (Q) → (3), (R) → (2), (S) → (1)

(D) (P) → (5), (Q) → (4), (R) → (1), (S) → (2)

Ans. **(A)**

Sol. $M=\begin{bmatrix}\alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}}\\ \gamma & \delta & \mu\end{bmatrix}$

$\vec{u}=\alpha\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\gamma\hat{k}$

$\vec{v}=\frac{1}{\sqrt{2}}\hat{i}+\beta\hat{j}+\delta\hat{k}$

$w=-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\mu\hat{k}$

$\alpha^2+\frac{1}{3}+\gamma^2=1 \quad \therefore\ \alpha^2+\gamma^2=\frac{2}{3} \quad \ldots(1)$

$\frac{1}{2}+\beta^2+\delta^2=1 \qquad \beta^2+\delta^2=\frac{1}{2} \quad \ldots(2)$

$\frac{1}{2}+\frac{1}{3}+\mu^2=1 \quad \mu^2=\frac{1}{6} \quad \ldots(3)$

$\gamma^2+\delta^2+\mu^2=1$

$\gamma^2+\delta^2=1-\frac{1}{6}=\frac{5}{6}$

For Q

$x\vec{u}+y\vec{v}+z\vec{w}=\hat{j}$

Dot with $x\ \vec{u}.\vec{u}+y\,\vec{u}.\vec{v}+z\vec{u}-\vec{w}=\vec{u}.\hat{j}$

$x=\frac{1}{\sqrt{3}}$

For R

$\vec{u},\vec{v},\vec{w}$ are mutually perpendicular unit vectors

$\left|\vec{u}.\left(\vec{v}\times\vec{w}\right)\right|$

$\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right]=1$

For S

$\vec{u}\times\left(\vec{v}\times\vec{w}\right)$

$\left(\vec{u}.\vec{w}\right)\vec{v}-\left(\vec{u}.\vec{v}\right)\vec{w}$

$=0-0\ =0$

16. Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List – I		**List – II**
(P)	The circle with centre (1,2) and touching the straight line $3x + 4y = 1$, passes through	(1)	the point (1,1)
(Q)	The common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ with positive slope, passes through	(2)	the point (7,9)
(R)	Let M be the end point of the latus rectum of the ellipse $3x^2 + 4y^2 = 48$ such that M lies in the first quadrant. Then the normal to the ellipse drawn at M passes through	(3)	the point (3, 2)
(S)	Let H be the hyperbola whose centre is at the origin, one of the foci is at (5, 0), and one directrix is $5x + 16 = 0$. Then H passes through	(4)	the point (2, 5)
		(5)	the point $\left(8, 3\sqrt{3}\right)$

(A) (P) → (3), (Q) → (4), (R) → (1), (S) → (2)

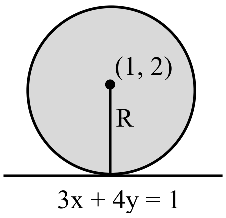
(B) (P) → (3), (Q) → (2), (R) → (1), (S) → (5)

(C) (P) → (3), (Q) → (2), (R) → (4), (S) → (5)

(D) (P) → (4), (Q) → (1), (R) → (2), (S) → (3)

Ans. **(B)**

Sol. (P)



R = ⊥ distance from (1,2) to line

$\therefore\ R = \dfrac{3+8-1}{5} = 2$

$\therefore$ Circle $(x-1)^2 + (y-2)^2 = 4$

point (3,2) satisfy

(Q) $y^2 = 4x,\ x^2 + y^2 = 2$

Let equation of tangent for $y^2 = 4x$

$\therefore\ y = mx + \frac{2}{m}$

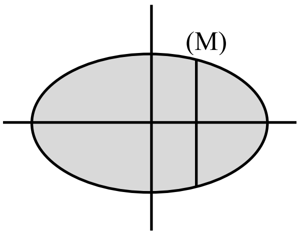
It is also a tangent for $x^2 + y^2 = 2$

So $\left|\frac{2/m}{\sqrt{1+m^2}}\right| = \sqrt{2}$ (condition of tangency)

if m = + 1

$\therefore$ equation of tangent = y + x + 2

so (7,9) satisfy

(R) 

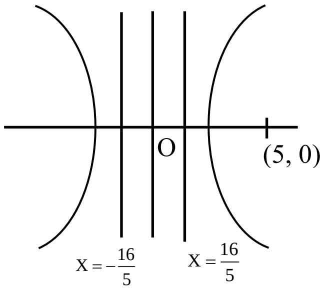
$\frac{x^2}{16} + \frac{y^2}{12} = 1 \qquad \therefore\ e = \frac{1}{2}$

$M\left(ae, \frac{b^2}{a}\right) = \left(4\times\frac{1}{2}, \frac{12}{4}\right) = (2,3)$

Equation of normal at (2,3) is

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2 \Rightarrow 2x - y = 1$

which passes through (1,1)

(S) 

$ae = 5$ and $ae = \frac{16}{5}$

$\therefore\ a = 4,\ e = \frac{5}{4}$

$\therefore\ \frac{x^2}{16} - \frac{y^2}{9} = 1$ is a hyperbola

which passes through $\left(8, 3\sqrt{3}\right)$