JEE(ADVANCED)–2026 (EXAMINATION)

(Held On Sunday 17th MAY, 2026)

MATHEMATICS | **TEST PAPER WITH ANSWER AND SOLUTION**

PAPER-2

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. Let $\vec{a},\vec{b}$ be two vectors, and let P, Q and R be the points with position vectors $\vec{a},\vec{b}$ and $\vec{a}+\vec{b}$, respectively, with respect to the origin O. If $|\vec{a}+\vec{b}|=\sqrt{21}$, $|\vec{a}-\vec{b}|=3$, and $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular to each other, then the area of the triangle OPR is

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{3}{2}$

Ans. **(C)**

Sol. $|\vec{a}+\vec{b}|=\sqrt{21}$; $|\vec{a}-\vec{b}|=3$

square & add

$2\left(|\vec{a}|^2+|\vec{b}|^2\right)=30 \Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2=15$

square & subtract

$4\vec{a}\cdot\vec{b}=12 \Rightarrow \vec{a}\cdot\vec{b}=3$

$\vec{a}\cdot(\vec{a}-\vec{b})=0 \Rightarrow |\vec{a}|^2=\vec{a}\cdot\vec{b}=3$

$|\vec{a}|^2=3$; $|\vec{b}|^2=12$

ar $(\Delta OPR)=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{3\sqrt{3}}{2}$

2. Let T be the tangent to the parabola $y^2 = 16x$ at the point (64, 32). Let L be the tangent to the same parabola at another point (x_1,y_1) on the parabola. If L and T are perpendicular to each other, then the distance between the point (x_1, y_1) and the focus of the parabola, is

(A) $\frac{15}{4}$ (B) 4

(C) $\frac{17}{4}$ (D) 5

Ans. **(C)**

Sol. $y^2 = 16x$; $(x_2, y_2) = (64, 32)$

$yy_2 = 8(x + x_2) \Rightarrow 32y = 8(x + 64)$

$x - 4y + 64 = 0$ (T)

Tangent L is $\perp$ to tangent T

Slope of line L $= -4 = \frac{1}{t} \Rightarrow t = -\frac{1}{4}$

$P = (x_1, y_1) = (4t^2, 8t) = \left(\frac{1}{4}, -2\right)$

Focus $= (4, 0) = F$

$FP = \sqrt{\frac{225}{16} + 4} = \sqrt{\frac{289}{16}} = \frac{17}{4}$

3. Let $y : (-\infty, \infty) \to (0, \infty)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{e^{5x}y^3 + y^3}{e^x + e^x y^4}$$

satisfying $y(0) = \frac{1}{\sqrt{2}}$. Then the value of $y(\log_e 2)$ is

(A) $\sqrt{\frac{5+\sqrt{35}}{2}}$ (B) $\sqrt{\frac{7+\sqrt{53}}{2}}$

(C) $\frac{7+\sqrt{53}}{2}$ (D) $\frac{5+\sqrt{35}}{2}$

Ans. **(B)**

Sol. $\frac{dy}{dx} = \frac{y^3(e^{5x}+1)}{e^x(1+y^4)}$

$$\int \frac{1+y^4}{y^3}dy = \int \frac{e^{5x}+1}{e^x}dx$$

$$-\frac{1}{2y^2} + \frac{y^2}{2} = \frac{e^{4x}}{4} - e^{-x} + C$$

$\downarrow \left(0, \frac{1}{\sqrt{2}}\right)$

$C = 0$

$$-\frac{1}{2y^2} + \frac{y^2}{2} = \frac{e^{4x}}{4} - e^{-x}$$

$\downarrow$ Put $x = \log_e 2$

$$-\frac{1}{2y^2}+\frac{y^2}{2}=4-\frac{1}{2}$$

Put $y^2=t$

$$\frac{-1+t^2}{2t}=\frac{7}{2}$$

$$t=\frac{7+\sqrt{53}}{2}=y^2$$

$$y=\sqrt{\frac{7+\sqrt{53}}{2}}$$

4. The value of the definite integral

$$\int_0^2 \frac{1}{3^x+3}dx \text{ is}$$

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{\log_e 3}{3}$

(D) $\frac{\log_e 3}{2}$

Ans. **(B)**

Sol. $\int_0^2 \frac{dx}{3^x+3}$;

$\int_0^2 \frac{3^{-x}dx}{1+3.3^{-x}}$; put $t=3^{-x}$

$$\frac{dt}{dx}=-3^{-x}\log_e 3$$

$$\int_1^{\frac{1}{9}} \frac{-dt}{\log_e 3\,(1+3t)}=\frac{1}{\log_e 3}\int_{\frac{1}{9}}^{1}\frac{dt}{1+3t}$$

$$\Rightarrow \frac{1}{3\log_e 3}\left(\log_e(1+3t)\right)_{\frac{1}{9}}^{1}$$

$$\frac{1}{3\log_e 3}\left(\log_e 4-\log_e\frac{4}{3}\right)=\frac{1}{3}$$

SECTION-2 : (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks ; and
  choosing any other combination of option(s) will get –1 marks.

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5. Let $\mathbb{R}$ denote the set of all real numbers. Consider the polynomial function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{d^{10}}{dx^{10}}((x^2 - 1)^{10}), \text{ for all } x \in \mathbb{R}$$

Here $\frac{d^{10}}{dx^{10}}((x^2-1)^{10})$ is the 10^{th} order derivative of the function $(x^2 - 1)^{10}$

Then which of the following statements is (are) TRUE ?

(A) The coefficient of x^8 in the polynomial f(x) is $(-10)\left(\frac{18!}{8!}\right)$

(B) The value of $f(1) + f(-1)$ is equal to $10!\ 2^{11}$

(C) The degree of the polynomial $f(x)$ is 10

(D) The constant term of the polynomial $f(x)$ is $-\left(\frac{10!}{5!}\right)$

Ans. **(A,B,C)**

Sol. $P(x) = (x^2 - 1)^{10}$

$\Rightarrow$ Degree = 20

$f(x) = P^{10}(x)$

Option C

when we differentiate 10-times ; Degree of f(x) = 20 – 10 = 10

$$P(x) = \sum_{r=0}^{10} {}^{10}C_r x^{20-2r}(-1)^r$$

Option A

$20 - 2r - 10 = 8 \Rightarrow r = 1$

$$f(x) = \ldots \underbrace{-{}^{10}C_1.(18.17.16\ldots 9)}_{-{}^{10}C_1.\frac{18!}{8!}}.x^8$$

Option D

$20 - 2r - 10 = 0 \Rightarrow r = 5$

$f(x) = \ldots -{}^{10}C_5\,(10.9.8 \ldots 3.2.1)$

$= -{}^{10}C_5.10!$

Option B

$P(x) = (x-1)^{10}(x+1)^{10}$

$P^{10}(1) = 2^{10}.10! = f(1)$

$P^{10}(-1) = 10!.2^{10} = f(-1)$

$f(1) + f(-1) = 2^{11}.10!$

6. Let a,b,c be positive integers in arithmetic progression such that the equation

$ax^2 + bx + c = 0$

has only integer solutions.

Then which of the following statements is (are) TRUE ?

(A) c – b is an integer multiple of a

(B) Both the roots of the equation $ax^2 + bx + c = 0$ are odd integers

(C) If c = 15, then ab = 8

(D) If b = 8, then x = 3 is a root of the equation $ax^2 + bx + c = 0$

Ans. **(A,B,C)**

Sol. a,b,c $\to$ AP ; 2b = a + c

$ax^2 + bx + c = 0$ has only integer solution α, β

$ax^2 + bx + c = a(x-\alpha)(x-\beta)$

put x = –2

$(\alpha + 2)(\beta + 2) = 3$

If a, b, c > 0 $\Rightarrow \alpha, \beta < 0$

$(\alpha + 2)(\beta + 2) = -1 \times -3$

$= -3 \times -1$

$(\alpha, \beta) = (-3, -5)\ ;\ (-5, -3)$

b = 8a ; c = 15a

Now verify the options

7. Let L be the straight line joining the points P(1,2, – 1) and Q(2,3,1). Let S be the foot of the perpendicular drawn from the point R(4, – 1, 5) to the line L. Another line passing through R intersects L at a point T such that the point S divides the line segment PT internally in the ratio $|PS|:|ST| = 1:2$, where $|PS|$ and $|ST|$ are the lengths of the line segments PS and ST, respectively.

Then which of the following statments is (are) TRUE ?

(A) The orthocentre of the triangle PRT is $\left(\frac{23}{5}, -4, \frac{31}{5}\right)$

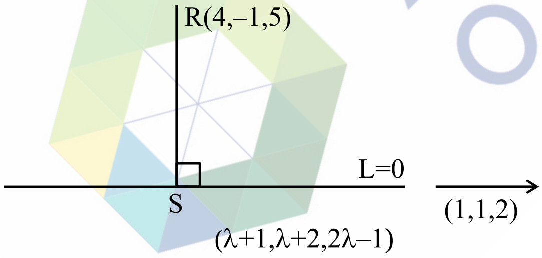
(B) The orthocentre of the traiangle PRT is (4,3,5)

(C) The area of the triangle PRT is $6\sqrt{5}$

(D) The area of the triangle PRT is $18\sqrt{5}$

Ans. **(A,D)**

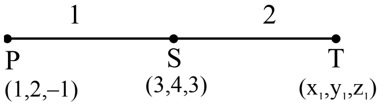
Sol. $L : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z+1}{2}$



$\overrightarrow{RS}.\left(\hat{i} + \hat{j} + 2k\right) = 0 \Rightarrow (\lambda - 3).1 + (\lambda + 3).1 + (2\lambda - 6).2 = 0$

$\Rightarrow \lambda = 2$

$\Rightarrow$ S (3, 4, 3)

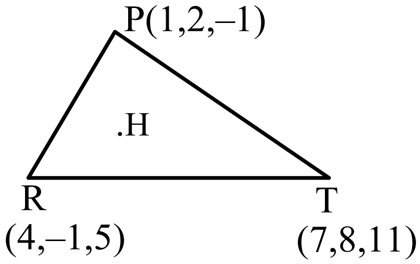


$$\frac{x_1+2}{3}=3 \Rightarrow x_1=7$$

$$\frac{y_1+4}{3}=4 \Rightarrow y_1=8$$

$$\frac{z_1-2}{3}=3 \Rightarrow z_1=11$$

$\Rightarrow T(7, 8, 11)$



Area of $\Delta PRT = \frac{1}{2}\left|\overrightarrow{RP}\times\overrightarrow{RT}\right|$

$\overrightarrow{RP} = -3\hat{i}+3\hat{j}-6\hat{k}$

$\overrightarrow{RT} = 3\hat{i}+9\hat{j}+6\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & -6 \\ 3 & 9 & 6 \end{vmatrix} = \hat{i}(18+54)-\hat{j}(0)+\hat{k}(-27-9)$$

$$= 72\hat{i}-36\hat{k}$$

$$= 36\left(2\hat{i}-\hat{k}\right)$$

$\Rightarrow$ Area of $\Delta PRT = \frac{1}{2}\times 36\sqrt{5} = 18\sqrt{5}$

Since $\overrightarrow{RS}\perp\overrightarrow{PT}$ so

Ortho centre H must lie on line RS.

also $\overrightarrow{PH}\perp\overrightarrow{RT}$

$H(4-\mu, -1+5\mu, 5-2\mu)$

$\overrightarrow{PH}.\overrightarrow{RT} = 0 \Rightarrow \mu = -3/5$

$\because H\left(\frac{23}{5}, -4, \frac{31}{5}\right)$

8. Let $y = f(x)$ be the real valued function defined on the interval $(0,\infty)$, satisfying $y(1) = 0$ and the differential equation

$$x\frac{dy}{dx} = y - x^3$$

Then which of the following statements is (are) TRUE ?

(A) The function f has a local minimum at $x = \frac{1}{\sqrt{3}}$

(B) The function f has a local maximum at $x = \frac{1}{\sqrt{3}}$

(C) The function f is increasing in the interval (1,2)

(D) If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for $x > 0$, then the number of elements in the set $\{x \in (0, \infty) : f(x) = g(x)\}$ is 2

Ans. **(B,D)**

Sol. $xdy - ydx = - x^3dx$

$$\frac{xdy - ydx}{x^2} = -xdx$$

$$d\left(\frac{y}{x}\right) = -xdx$$

$$\Rightarrow \frac{y}{x} = \frac{-x^2}{2} + c$$

$$y = \frac{-x^3}{2} + cx$$

As $y(1) = 0 \Rightarrow c = \frac{1}{2}$

$$y = \frac{-x^3}{2} + \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{-3x^2}{2} + \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{-3}{2}\left(x - \frac{1}{\sqrt{3}}\right)\left(x + \frac{1}{\sqrt{3}}\right)$$

$-$ $\quad$ $-1/\sqrt{3}$ $\quad$ $+$ $\quad$ $1/\sqrt{3}$ $\quad$ $-$

Local maxima at $x = \frac{1}{\sqrt{3}}$

Local minima at $x = \frac{-1}{\sqrt{3}}$

For $g(x) = f(x)$

$4x^3 - 5x^2 + \frac{3}{2}x = \frac{-x^3}{2} + \frac{x}{2}$

$9x^3 - 10x^2 + 2x = 0$

$x(9x^2 - 10x + 2) = 0$

$x = 0, \quad x = \frac{5 \pm \sqrt{7}}{9}$ (Both are positive)

9. Let $\mathbb{R}$ denote the set of all real numbers and let $i = \sqrt{-1}$. Consider the matrices

$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Let a, b, c, d be real numbers such that

$ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Let

$H = \{x + iy : \quad x, y \in \mathbb{R} \text{ and } y > 0\}$.

Then which of the following statements is (are) TRUE?

(A) $\frac{b + ia}{d + ic} = i$

(B) If $\omega = \frac{-1 + i\sqrt{3}}{2}$, then $\frac{a\omega + b}{c\omega + d} = \omega$

(C) If m is an integer greater than 2 such that $(ST)^2 = (ST)^m$, then m is an integer multiple of 8

(D) If $z \in H$, then $\frac{az + b}{cz + d} \in H$

Ans. **(B,D)**

Sol. $ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a = 0, b = -1, c = 1, d = 1$

(A) $\frac{b+ia}{d+ic} = \frac{-1}{1+i} \times \frac{1-i}{1-i} = \frac{-1+i}{2}$ (incorrect)

(B) $\frac{a\omega+b}{c\omega+d} = \frac{-1}{\omega+1} = \frac{-1}{-\omega^2} = \omega$ (correct)

(C) $ST^2 = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$

$ST^3 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

$(ST)^6 = I$

$(ST)^2 = (ST)^M \Rightarrow M = 6\lambda + 2; \lambda \in I$ (incorrect)

(D) $\frac{az+b}{cz+d} = \frac{-1}{z+1}$

$= \frac{-1}{x+iy+1} = \frac{-1}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$= \frac{-(x+1)+iy}{(x+1)^2+y^2}$

Imaginary part $= \frac{y}{(x+1)^2+y^2} > 0$ as $y > 0$ is given

SECTION-3 : (Maximum Marks : 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

10. Let $\mathbb{N}$ denote the set of all positive integers. Consider the sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{1, 2, 3, 4, 5, 6, 7\}.$$

Let S be the set of all functions $f: A \to B$ such that $f(2) \neq 2$ and $f(4) \neq 4$. Consider the set

$T = \{f \in S : \text{there exists a function } g : B \to \mathbb{N} \text{ such that } g(f(x)) = 2^x \text{ for all } x \in A\}$.

Then the number of elements in the set T is ___________.

Ans. **1860.00**

Sol. As $g(f(x)) = 2^x\ \forall x \in A$

so f(x) must be one-one function.

Total no. of functions = ${}^7C_5 \times 5! = 2520$

Regd. no. of functions = Total – (fn. with f(2) = 2 + fn. with f(4) = 4) + (fn. with f(2) = 2 & f(4) = 4)

$= 2520 - ({}^6C_4 \times 4! + {}^6C_4 4!) + {}^5C_2 \times 3!$

$= 2520 - (360 + 360) + 60$

$= 1860$ Ans.

11. A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let X be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If α is the mean of the random variable X, then the value of 77α is _________.

Ans. **100.00**

Sol. Let no. of maths books selected = a

and Physics Books selected = b

$a + b = 6$ & $X = |a - b|$

$X = 6 \Rightarrow a = 6, b = 0$

$$P(X = 6) = \frac{{}^6C_6 \cdot {}^5C_0}{{}^{11}C_6} = \frac{1}{{}^{11}C_6}$$

$X = 4 \Rightarrow a = 5, b = 1$ or $a = 1, b = 5$

$$P(X = 4) = \frac{{}^6C_5.{}^5C_1 + {}^6C_1.{}^5C_5}{{}^{11}C_6} = \frac{36}{{}^{11}C_6}$$

$X = 2 \Rightarrow a = 2, b = 4$ or $a = 4, b = 2$

$$P(X = 2) = \frac{{}^6C_2.{}^5C_4 + {}^6C_4.{}^5C_2}{{}^{11}C_6} = \frac{225}{{}^{11}C_6}$$

$X = 0 \Rightarrow a = 3, b = 3$

$$P(X = 0) = \frac{{}^6C_3.{}^5C_3}{{}^{11}C_6} = \frac{200}{{}^{11}C_6}$$

$$\text{Mean} = \sum X.P(X)$$

$$\alpha = 6\times\frac{1}{{}^{11}C_6} + 4\times\frac{36}{{}^{11}C_6} + 2\times\frac{225}{{}^{11}C_6} + 0\times\frac{200}{{}^{11}C_6}$$

$$\alpha = \frac{100}{77}$$

$\Rightarrow 77\,\alpha = 100$

12. Consider a data consisting of 10 observations $x_1, x_2, \ldots, x_{10}$, whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations $x_1, x_2, \ldots, x_8$ are 4 and 3.5, respectively, and $x_9 < x_{10}$, then the value of $3x_9 + 2x_{10}$ is____________.

Ans. **44.00**

Sol. $\sum_{i=1}^{10} x_i = 50$

$$7 = \frac{\sum_{i=1}^{10} x_i^2}{10} - 25 \Rightarrow \sum_{i=1}^{10} x_i^2 = 320$$

$$\text{Again, } 3.5 = \frac{\sum_{i=1}^{8} x^2}{8} - 16 \Rightarrow \sum_{i=1}^{8} x_i^2 = 156$$

$$\sum_{i=1}^{8} x_i = 32 \Rightarrow x_9 + x_{10} = 18$$

$$x_9^2 + x_{10}^2 = 320 - 156 = 164$$

$$x_9^2 + (18 - x_9)^2 = 164 \Rightarrow 2x_9^2 - 36\,x_9 + 160 = 0$$

$\Rightarrow x_9^2 - 18x_9 + 80 = 0$

$\Rightarrow x_9 = 10$ or 6

$\Rightarrow x_9 = 8, x_{10} = 10$

$\therefore 3x_9 + 2.x_{10} = 24 + 20 = 44$

13. Consider the ellipse E given by $\frac{x^2}{18}+\frac{y^2}{12}=1$. Let H be the hyperbola whose eccentricity is the reciprocal of the eccentricity of E and whose foci are the same as that of E. Let P and Q be the points of intersection of H and the parabola $\sqrt{5}\,y = x^2$ in the first quadrant. Let d be the distance between P and Q.

If a and b are the integers such that $d^2 = a + b\sqrt{5}$, then the value of $a - b$ is ___________.

Ans. **18.00**

Sol. $E \equiv \frac{x^2}{18}+\frac{y^2}{12}=1$

$e=\sqrt{1-\frac{12}{18}}=\frac{1}{\sqrt{3}}$

so $e_H=\sqrt{3}$

$ae=\sqrt{a^2-b^2}=\sqrt{18-12}=\sqrt{6}$

foci are $\left(\pm\sqrt{6},0\right)$

$(Ae_H)^2 = A^2 + B^2 = 6$

$Ae_H=\sqrt{6}$

$A=\sqrt{2}$

$B^2 = 4$

Thus $H \equiv \frac{x^2}{2}-\frac{y^2}{4}=1$

Solving with $x^2=\sqrt{5}y$

$\frac{\sqrt{5}\,y}{2}-\frac{y^2}{4}=1$

y_1 and y_2 are roots of $y^2-2\sqrt{5}y+4=0$

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ then

$(PQ)^2 = d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$|y_1-y_2|=\frac{\sqrt{20-16}}{1}=2$

$d^2 = x_1^2 + x_2^2 - 2x_1x_2 + 4$

$= \sqrt{5}y_1 + \sqrt{5}y_2 - 2\times 2\sqrt{5} + 4$

$= \sqrt{5}\left(2\sqrt{5}\right) - 4\sqrt{5} + 4$

$d^2 = 14 - 4\sqrt{5}$

$a = 14$ & $b = -4$

$a - b = 18$

14. For a real number α, let $[\alpha]$ denote the greatest integer less than or equal to α. For a finite set S, let $|S|$ denote the number of elements in the set S.

Consider the functions $f: (-3,3)\rightarrow(-\infty,\infty)$ and $g: (-3,3) \rightarrow (-\infty,\infty)$ defined by

$f(x) = [x^3]\log_e(1 + \sin^2(\pi(x - [x])))$

and $(x) = x^3 \sin^2 (\pi \log_e(1 + x - [x]))$.

Let $A = \{x \in (-3, 3) : f \text{ is discontinuous at } x\}$

and $B = \{x \in (-3, 3) : g \text{ is discontinuous at } x\}$.

Then the value of $|A| + 2|B| - |A \cap B|$ is ________.

Ans. **56.00**

Sol. $f(x) = [x^3]\log(1+\sin^2\pi(x - [x]))$

$f(x) = [x^3]\log(1 + \sin^2\pi x)$

$x \in (-3, 3)$

$x^3 \in (-27, 27)$

At integers $x = \pm1, \pm2, 0$

$f(x)$ is continuous as $\log(1 + \sin^2\pi x) = 0$

So number of points of discontinuity of $f(x)$ is $53 - 5 = 48$

and $g(x) = x^3\sin^2\pi\log_e(1+\{x\})$ is cont. at $x = 0$,

So point of Discontinuity are $x = \pm1$ & ±2

$|A| = 48$ & $|B| = 4$ & $|A \cap B| = 0$

$|A| + 2|B| - |A \cap B|$

$= 48 + 8 - 0 = 56$

SECTION-4 : (Maximum Marks : 8)

- This section contains **TWO (02)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +2 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 15 and 16

Consider the curve C_1 given by

$$y = e^{-x} \text{ for } x \in [0, 10\pi],$$

and the curve C_2 given by

$$y = e^{-x}(\sin x + \cos x) \text{ for } x \in [0, 10\pi].$$

Let n be the total number of points of intersection of the curves C_1 and C_2.
Suppose that $\alpha_1, \alpha_2, \ldots, \alpha_n \in [0, 10\pi]$ are the x - coordinates of the points of intersection of the curves C_1 and C_2 such that

$$\alpha_1 < \alpha_2 < \ldots < \alpha_n.$$

15. The value of n is ___________.

Ans. **11.00**

Sol. $e^{-x} = e^{-x}(\sin x + \cos x)$

$\sin x + \cos x = 1$

$$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$x = 2n\pi \text{ or } 2n\pi + \frac{\pi}{2}$$

$$x \in \left\{0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, 4\pi, \frac{9\pi}{2}, 6\pi, \frac{13\pi}{2}, 8\pi, \frac{17\pi}{2}, 10\pi\right\}$$

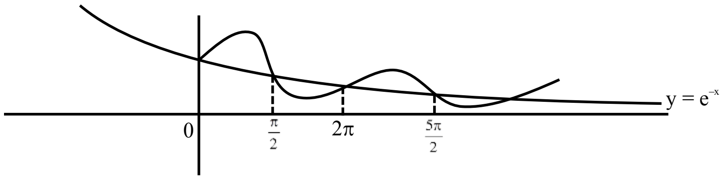
so $n = 11$

16. Let β be the area of the region enclosed between the curves C_1, C_2, and the lines $x = \alpha_1$ and $x = \alpha_4$. Then the value of

$$-\frac{1}{\pi}\log_e\left(\beta - 2e^{-\frac{\pi}{2}}\right) \text{ is } \_\_\_\_\_\_\_\_\_\_.$$

Ans. **2.50**

Sol. $\alpha_1 = 0 \ \& \ \alpha_4 = \frac{5\pi}{2}$



$$\beta = \int_0^{\frac{5\pi}{2}} \left|\left(e^{-x}(\sin x + \cos x) - e^{-x}\right)\right| dx$$

$$\beta = \int_0^{\frac{\pi}{2}} \left(e^{-x}(\sin x + \cos x) - e^{-x}\right) dx + \int_{\frac{\pi}{2}}^{2\pi} \left(e^{-x} - e^{-x}(\sin x + \cos x)\right) dx + \int_{2\pi}^{\frac{5\pi}{2}} \left(e^{-x}(\sin x + \cos x) - e^{-x}\right) dx$$

$$\beta = 2e^{\frac{-\pi}{2}} + e^{\frac{-5\pi}{2}}$$

so $\frac{-1}{\pi}\log\left(\beta - 2e^{\frac{-\pi}{2}}\right) = \frac{5}{2} = 2.50$

Question Stem for Question Nos. 17 and 18

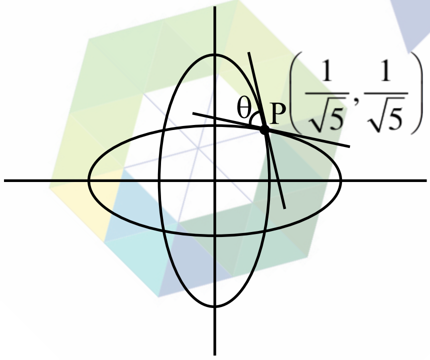
Consider the ellipses given by

$$x^2 + 4y^2 = 1 \text{ and } 4x^2 + y^2 = 1.$$

17. Let P be the point in the first quadrant where the given ellipses intersect. If θ is the acute angle between the tangents to the given ellipses at the point P, then the value of $4 \tan\theta$ is ____________.

Ans. **7.50**

Sol.



$x^2 + 4y^2 = 1$

Diff. w.r.t. x

$2x + 8yy' = 0$

$y' = \frac{-x}{4y}$

$m_1 = \frac{-1}{4}$

Similarly $4x^2 + y^2 = 1$

$8x + 2yy' = 0$

$y' = \frac{-4x}{y}$

$m_2 = -4$

$\tan\theta = \left|\frac{\frac{-1}{4}+4}{1+1}\right|$

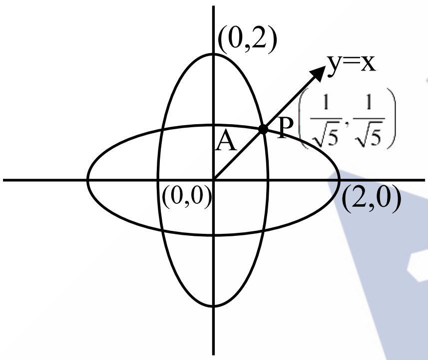
$\tan\theta = \frac{15}{8}$

$4\tan\theta = \frac{15}{2} = 7.50$

18. If α is the area of the common region that lies inside both the given ellipses, then the value of cot α is ___________.

Ans. **0.75**

Sol.



$A = \int_0^{\frac{1}{\sqrt{5}}}\left(\frac{\sqrt{1-x^2}}{2} - x\right)dx$

Total required area

$\alpha = 8A = 8\left[\int_0^{\frac{1}{\sqrt{5}}}\left(\frac{\sqrt{1-x^2}}{2} - x\right)dx\right]$

$\alpha = 8\left[\frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1}x - \frac{x^2}{2}\right]_0^{\frac{1}{\sqrt{5}}}$

$\alpha = 8\left[\frac{1}{4\sqrt{5}}\sqrt{\frac{4}{5}} + \frac{1}{4}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{1}{10}\right]$

$$\alpha = 2\sin^{-1}\frac{1}{\sqrt{5}}$$

$$\cot\alpha = \cot\left(2\sin^{-1}\frac{1}{\sqrt{5}}\right)$$

$$= \cot\left(2\tan^{-1}\frac{1}{2}\right)$$

$$= \cot\left(\tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)\right)$$

$$= \cot\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$$

$$\cot\alpha = \frac{3}{4} = 0.75$$